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**DIGITAL COMPUTER ORIENTED METHODS  
FOR DETERMINING THE RESPONSE  
OF PRESSURE MEASUREMENT SYSTEMS  
TO STEP AND RAMP FORCING FUNCTIONS**

**M. L. McKee**

**ARO, Inc.**

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**March 1967**

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# ERRATA

AEDC-TR-66-225, March 1967  
(UNCLASSIFIED REPORT)

## DIGITAL COMPUTER ORIENTED METHODS FOR DETERMINING THE RESPONSE OF PRESSURE MEASUREMENT SYSTEMS TO STEP AND RAMP FORCING FUNCTIONS

M. L. McKee, ARO, Inc.

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pp. 3 & 51: Eqs. (2) and (III-15) should read:

$$K_m = 5.16 \times 10^{-6} \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{l_1}{d_1^4} \right) \left\{ 2A_1 l_1 \right. \right. \\ \left. \left. + A_2 l_2 (3D + 6) + A_3 l_3 (3D + 3B + 6) + 6V(B + 1) \right\} \right. \\ \left. + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{l_2}{d_2^4} \right) \left\{ A_2 l_2 (3C + 1) + A_3 l_3 (C^2 + 3C) \right\} \right. \\ \left. + \left( \frac{2}{C+1} \right) \left( \frac{l_2}{d_2^4} \right) \left\{ A_2 l_2 + A_3 l_3 (C^2 + 3C + 3) + 3V(C + 1)^2 \right\} \right]$$

pp. 4, 46 & 51: Eqs. (5), (II-39) and (III-16) should read:

$$t_L = \frac{K_m}{2(P_i + \delta)} \log_e \left| \frac{2\delta(P_i + \delta) - |\delta e|}{e(2P_i + \delta)} \right|$$

pp. 4 & 54: Eqs (3) and (IV-17) should read:

$$K_T = 5.16 \times 10^{-6} \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{l_1}{d_1^4} \right) \left\{ A_1 l_1 (3B + 1) \right. \right. \\ \left. \left. + A_2 l_2 (3B + 3DC) + A_3 l_3 (3DC) \right\} \right. \\ \left. + \left( \frac{B}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{l_2}{d_2^4} \right) \left\{ A_2 l_2 (3C + 1) + A_3 l_3 (C^2 + 3C) \right\} \right]$$

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pp. 18-24: Fig. 3 Generalized Plots of Program Calculated Pressure Lag Times-Step Forcing Function

Units for  $t_L/K^*$  are sec.

Units for  $t_L/K^*$  should be  $\text{psf}^{-1}$ .

pp. 20-24: Fig. 3 Generalized Plots of Program Calculated Pressure Lag Times-Step Forcing Function

The parameter for each curve should be  $\Delta p$  in psf.

p. 44: Eq. (II-23) should read:

$$\begin{aligned} K_T = & \frac{8\pi\mu}{3} \left\{ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{A_1^2} \right) \left[ A_1 \ell_1 (3B+1) + A_2 \ell_2 (3B+3DC) \right. \right. \\ & + \left. A_3 \ell_3 (3DC) \right] + \left( \frac{B}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{A_2^2} \right) \left[ A_2 \ell_2 (3C+1) \right. \\ & \left. \left. + A_3 \ell_3 (C^2 + 3C) \right] \right\} \end{aligned}$$

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M. L. McKee  
ARO, Inc.

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## FOREWORD

The work reported herein was done at the request of Headquarters, Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC) under Program Element 65402234.

The results of research presented were obtained by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee, under Contract AF 40(600)-1200. The research was conducted from January to May, 1965, under ARO Project No. PT8002, and the manuscript was submitted for publication on October 18, 1966.

This technical report has been reviewed and is approved,

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## ABSTRACT

Theoretical equations for unsteady, compressible, laminar flow in cylindrical tubing have been derived for pressure measuring systems comprised of up to three tube sizes connected to constant volume transducers. The basic equations have been solved for both step and ramp pressure input forcing functions. Digital computer programs have been written to be used as engineering tools for rapid determination of system characteristics. Examples of the ability of the programs to compare tubing systems, optimize diameters, generate time responses to synthesized forcing functions, and determine lag times, distortion, and attenuation are presented as guides for prospective users of the programs.





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## NOMENCLATURE

A	Internal cross-sectional area of tube, $\text{ft}^2$
a	Coefficient in Weber's equation, elsewhere the dimensionless condition for neglecting acceleration forces
B	Dimensionless geometric ratio
C	Dimensionless geometric ratio, or a constant when accompanied by subscript
D	Dimensionless geometric ratio

d	Internal diameter of tube, ft
e	Natural base, 2.7183
F(p)	Function of pressure only, $\text{psf}^{-1}$
f	Frequency, cps
f(t)	Pressure forcing function, $\text{psf}/\text{sec}$
g(t)	Transformation function
i	$\sqrt{-1}$
K	Constants as defined
$K_m, K_T$	Geometric characteristics of tubing system, $\text{psf}\cdot\text{sec}$
$K^*$	$K_m \cdot 10^{-3}$
k	Forcing function coefficient, $\text{psf}/\text{sec}^3$
$\ell$	Length of tube, ft
m	Forcing function coefficient, $\text{psf}/\text{sec}^2$
n	Forcing function coefficient, $\text{psf}/\text{sec}$
P	Transducer pressure, psf
$P_o$	Initial transducer pressure
p	General pressure, psf
R	Gas constant, $1715 \text{ ft}\cdot\text{lb}/\text{slug}\cdot^\circ\text{R}$
Re	Reynolds number
T	Temperature, $^\circ\text{F}$
$T_1$	Temperature, $^\circ\text{R}$
$T_r$	Reference temperature, $524.4 \text{ }^\circ\text{R}$
t	Time, sec
$t_L$	Time lag, sec
u	Known solution to Riccati equation, $\text{psf}^{-1}$
V	Transducer volume, $\text{ft}^3$
$W(a, x)$	Weber function
$W'(a, x)$	$\frac{d W(a, x)}{dx}$
w	Mass flow rate, $\text{slug}/\text{sec}$
x	Distance along tube, ft, or transformation variable as defined

$y$	Transformed pressure function
$\Gamma$	Gamma function
$\delta, \Delta p$	Amplitude of pressure change, psf
$\epsilon$	Error in final pressure, psf
$\mu$	Viscosity, slug/ft-sec
$\mu_r$	Reference viscosity, $3.8 \times 10^{-7}$ slug/ft-sec
$\pi$	3.1416
$\tau$	Period of forcing function sec,

### SUBSCRIPTS

1	First tube from orifice
2	Second tube from orifice
3	Third tube from orifice
12	Junction of first and second tubes
23	Junction of second and third tubes
avg	Average
comp	Compromise
f	Final
i	Initial or as otherwise specified
min	Minimum
opt	Optimum
t, orf	Orifice
trans	Transducer

## SECTION I

### INTRODUCTION

Measurement of a large number of test article pressures during wind tunnel testing invariably requires that the pressure transducers be located outside of the test cell because of the physical dimensions and environmental limitations of present day high accuracy pressure transducers. The result is that the long lengths of tubing that are required to connect model orifices to the transducers act as delay lines. If the orifice pressures are steady values, the pressures seen by the transducers will eventually become the true orifice values, but only after an interval of prime testing time, following a change in pressure conditions, has been expended. With the model orifice sizes and tubing configurations presently being used in the 16-ft wind tunnels (Propulsion Wind Tunnels, Transonic (16T) and Supersonic (16S)), it is not uncommon for from 10 to 20 percent of the prime on-line testing time available for a test program to be devoted to pressure stabilization time. Following a Mach number extension of the supersonic tunnel to be completed in the near future, this problem will become even more critical because of the accompanying lower pressures.

In view of this problem, a study was initiated to develop a flexible digital computer program that could be used as an engineering tool to:

1. Study the effects of all tubing dimensions upon the system response.
2. Determine approximate lag times for use in determining more realistic test schedules and minimum stabilization times.
3. Determine optimum tubing dimensions within given restraints.
4. Approximate transducer response to slowly varying orifice pressures, including complex orifice pressure-time relationships.
5. Compare relative time responses of different tubing configurations.

Several theoretical and experimental investigations of these problems as they pertain to wind tunnel instrumentation have been made. Bauer (Ref. 1) and Kinslow (Ref. 2) have made detailed studies of one- and two-tube pressure measuring systems of constant volume. However, since the model orifice diameters and lengths are generally fixed by the test sponsor and the transducers are installed with permanent tubing to a

disconnect panel, a three-tube system is usually employed in PWT. The equations of Refs. 1 and 2 are not sensitive to the order of the tubes in systems in which more than two tube sizes are used. The theoretical results of Refs. 1 and 2, therefore, have been extended to include three-tube systems in such a manner that the response is sensitive not only to the geometry, but also the order, of each tube. The resulting differential equation relating pressure and time has been solved for both step and ramp forcing functions.

The purpose of this report is to present the theoretical extension and to explain the utility and limitations of the resulting step and ramp computer programs. Examples are given to serve as guides in the employment of these engineering tools. Experimental verification of second-tube diameter optimization is also included.

## SECTION II SYSTEM EQUATIONS

The computer equations are based upon an extension of the theoretical analyses of unsteady, compressible, laminar flow in cylindrical tubing performed by Bauer (Ref. 1) and Kinslow (Ref. 2). Bauer considered the distribution of volume in a two-tube constant volume system but neglected the change in mass flow rate. Kinslow refined the theory by including the effect of variation of mass flow in a two-tube system. To account for multidiameter tubing configurations, Kinslow suggests replacement of all the tubes, with the exception of the tube next to the transducer, by an equivalent tube having the same volume, mass flow, and pressure drop as the tubes it replaces. The equations for determining the geometry of the equivalent tube, however, are derived assuming constant mass flow in the replaced tubes. Hence, the equivalent tube is not sensitive to the order of the replaced tubes. To properly study the flow, and in particular to optimize the diameter of any given tube in a multidiameter system, the intuitive conclusion is that the equations describing the flow must be sensitive not only to the geometry, but also to the order, of each tube.

The analysis is based upon the unsteady form of the Hagen-Poiseuille equation of laminar flow of a viscous fluid through a circular tube of constant diameter. The two-dimensional flow is assumed to possess axial symmetry, with the velocity distributed parabolically over the radius. The following assumptions also apply:

1. The process is isothermal.
2. Pressure and density are constant over any cross section of a tube.
3. Pressure and mass flow are constant across any junction of two tubes.
4. The changes in pressure and density along a tube are linear.
5. The times and lengths required for complete establishment of flow are small compared to the lag times and tubing lengths involved.
6. The fluid is a perfect gas.
7. Acceleration forces are negligible compared to viscous forces.
8. Outgassing and leakage rates are negligible.
9. Volume of tubing transducer system is constant.

In order to determine the time response of the transducer pressure, a differential equation relating the transducer pressure to the orifice pressure (forcing function) and the geometric dimensions of the system is required. The methods used to derive this equation for a three-tube system are similar to those used by Kinslow (Ref. 2) in deriving the two-tube equation. The pressure measurement system for which the equation was developed is shown schematically in Fig. 1 (Appendix I). A detailed derivation of the system equations is included in Appendix II. The system equations are:

#### DIFFERENTIAL EQUATION

$$P_t^2 - P^2 = K_T \frac{dP_t}{dt} + K_m \frac{dP}{dt} \quad (1)$$

#### GEOMETRIC CHARACTERISTICS

$$\begin{aligned}
 K_m = 5.16 \times 10^{-6} & \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{l_1}{d_1^4} \right) \left\{ 2A l_1 \right. \right. \\
 & + A_2 l_2 (3D + 6) + A_3 l_3 (3D + 3B + 6) + 6V(B + 1) \left. \right\} \\
 & + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{l_2}{d_2^4} \right) \left\{ A_2 l_2 (3C + 1) + A_3 l_3 (C^2 + 3C) \right\} \\
 & \left. + \left( \frac{2}{C+1} \right) \left( \frac{l_2}{d_2^4} \right) \left\{ A_2 l_2 + A_3 l_3 (C^2 + 3C + 3) + 3V(C + 1)^2 \right\} \right] \quad (2)
 \end{aligned}$$

$$K_T = 5.16 \times 10^{-6} \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{d_1^4} \right) \left\{ A_1 \ell_1 (3B + 1) \right. \right. \\ \left. \left. + A_2 \ell_2 (3B + 3DC) + A_3 \ell_3 (3DC) \right\} + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 (3C + 1) + A_3 \ell_3 (C^2 + 3C) \right\} \right] \quad (3)$$

$$K^* = K_m \cdot 10^{-3} \quad (4)$$

### STEP SOLUTION

$$t_L = \frac{K_m}{2(P_i + \delta)} \log_e \left| \frac{2\delta(P_i + \delta) - \delta\epsilon}{\epsilon(2P_i + \delta)} \right| \quad (5)$$

### RAMP SOLUTION

$$P = K_m (4k)^{1/4} \left\{ \frac{C_1 W(a, x) - W'(a, -x)}{C_1 W(a, x) + W(a, -x)} \right\} \quad (6)$$

## SECTION III APPROXIMATE RAMP SOLUTION

The ramp solution given by Eq. (6) is quite difficult to evaluate, even when using a digital computer. An approximate solution that is quite good for a great many cases has been found. Equation (1) can be rewritten as

$$P^2 = P_t^2 - K_m \left( 1 + \frac{K_T}{K_m} \right) \frac{dP_t}{dt} + K_m \left( \frac{dP_t}{dt} - \frac{dP}{dt} \right) \quad (7)$$

The third term on the right of Eq. (7) is the difference between the two rates of pressure change. In many systems, in particular a Type 1 control system, following a short stabilization period, the output will follow the input with equal rate but will have a finite difference in value as shown in Sketch 2, Appendix II. Hence, the third term on the right can be neglected in cases where the pressure measurement system-forcing function combination is such that the system appears as a Type 1 control system. This simplification can be explained in another way.



Equation (7) can be expanded in a binomial series as

$$\begin{aligned}
 P^2 = & P_t^2 - K_m \left( 1 + \frac{K_T}{K_m} \right) \frac{dP_t}{dt} - \frac{K_m^2}{2} \left( 1 + \frac{K_T}{K_m} \right) \left( \frac{1}{P_t} \frac{dP_t}{dt} \right)^2 \\
 & + \frac{K_m^2}{2} \left( \frac{dP_t}{dt} \right)^2 - \frac{K_m^2}{2} \left( \frac{1}{P_t} \right)^2 \left( \frac{dP}{dt} \frac{dP_t}{dt} \right) + \frac{K_m^2}{2} \left( \frac{1}{P} \right) \left( \frac{d^2P}{dt^2} + \frac{K_m}{K_T} \frac{d^2P_t}{dt^2} \right) + \text{higher order terms}
 \end{aligned} \quad (8)$$

Hence, neglecting the third term on the right of Eq. (7) is equivalent to neglecting all terms of order 2 and higher in the series expansion of  $P^2$ .

The approximate ramp solution is then given by

$$P^2 = \left\{ P_i + \left( \frac{P_f - P_i}{\tau} \right) t \right\}^2 - K_m \left( 1 + \frac{K_T}{K_m} \right) \left( \frac{P_f - P_i}{\tau} \right) \quad (9)$$

Once the values of  $K_m$  and  $K_T$  are known for a particular system, numerical values of  $P$  can be easily hand computed using Eq. (9).

#### SECTION IV QUALIFYING CONDITIONS

The applicability of Eq. (1) is contingent upon the validity of the assumptions that the flow is laminar and the acceleration forces are small. From Ref. 2, the Reynolds number condition for a single tube is given by

$$\left| \frac{d^3(P_t^2 - P^2)}{64\mu^2 l R T_1} \right| \leq 2000 \quad (10)$$

and the condition for neglecting acceleration is given by

$$\left| \frac{24(\ell/d)}{Re \log_e(P/P_t)} \right| \gg 1 \quad (11)$$

Expressing the junction pressures as

$$P_{12} = \left( \frac{B}{B+1} \right) (P_i + \delta) + \left( \frac{1}{B+1} \right) P_i \quad (12)$$

$$P_{23} = \left( \frac{DC}{B+1} \right) (P_i + \delta) + \left( \frac{D+1}{B+1} \right) P_i \quad (13)$$

the six qualifying conditions for the three-tube system are

$$\text{Re}_1 = \left| \frac{d_1^3}{64\mu^2 \ell_1 R T_1} (P_i + \delta)^2 - P_{12}^2 \right| \quad (14)$$

$$\text{Re}_2 = \left| \frac{d_2^3}{64\mu^2 \ell_2 R T_1} \left\{ P_{12}^2 - P_{23}^2 \right\} \right| \quad (15)$$

$$\text{Re}_3 = \left| \frac{d_3^3}{64\mu^2 \ell_3 R T_1} \left\{ P_{23}^2 - P^2 \right\} \right| \quad (16)$$

$$a_1 = \left| \frac{24 (\ell_1/d_1)}{\text{Re}_1 \log_e \left( \frac{P_{12}}{P_i + \delta} \right)} \right| \quad (17)$$

$$a_2 = \left| \frac{24 (\ell_2/d_2)}{\text{Re}_2 \log_e \left( \frac{P_{23}}{P_{12}} \right)} \right| \quad (18)$$

$$a_3 = \left| \frac{24 (\ell_3/d_3)}{\text{Re}_3 \log_e \left( \frac{P}{P_{23}} \right)} \right| \quad (19)$$

## SECTION V COMPUTER PROGRAMS

The equation lists used in writing the step and ramp computer programs are summarized, for convenience, in Appendixes III and IV, respectively.

### 5.1 STEP PROGRAM

The step program is primarily used to determine the lag time following a step change in orifice pressure. System dimensions, initial pressure (initial transducer pressure must be equal to initial orifice pressure), change in orifice pressure, and desired settling error are inputs to the program. The standard input sheet is shown in Fig. 2. The time lag and qualifying conditions are the outputs. The step program has been written in Fortran II for use in the PWT Raytheon 520 computer.

Since, in many cases, the pressure parameters are not known, a comparison of several possible tubing systems based upon geometry and fluid

properties alone is desirable. Applying the method of separation of variables, the lag time can be written as

$$t_L = K_m \cdot F(p) \quad (20)$$

where  $K_m$  is a function of geometry and fluid properties, whereas  $F(p)$  is a function of the pressure conditions alone. Since  $K_m$ , as given by Eq. (2), completely characterizes the system, comparison of different systems can be accomplished simply by comparison of the respective  $K_m$  values. A more convenient value of the geometric characteristic is  $K^*$ , where

$$K^* = K_m \cdot 10^{-3}$$

Generalized plots of  $F(p)^* = \frac{t_L}{K^*}$  for various values of  $P_i$ ,  $\delta$ , and  $\epsilon$  can be generated by the program. The actual lag time can then be determined from these plots for a particular system ( $i$ th system) with geometric characteristic  $K_i^*$  by

$$t_{L_i} = \frac{t_L}{K^*} \cdot K_i^* \quad (21)$$

Typical plots of  $t_L/K^*$  are shown in Fig. 3.

If the diameter and length of the first and third tube is either specified by test requirements or permanently fixed, as is usually the case in PWT, the optimum diameter of the middle, or connecting, tube is defined as the diameter that minimizes the value of  $K^*$ . The smaller the tube, the larger the flow resistance caused by viscosity of the fluid. As the tube becomes larger, the volume of fluid in the tube that must be moved is increased. Hence, a minimum value of  $K^*$  always exists. The computer is programmed to optimize the second tube of either a two- or three-tube system by iteration of the diameter in steps of 0.003 in. The output contains  $(d_2)_{opt}$ ,  $K^*_{min}$ , and the  $K^*$  values corresponding to  $d_2$  values both smaller and larger than  $(d_2)_{opt}$  by 0.003 and 0.006 in.

The six options available with the program are explained in Fig. 2. Typical option 1, 5, and 6 outputs are shown in Figs. 4, 5, and 6, respectively.

The viscosity is corrected for temperature using Sutherland's formula

$$\mu_1 = \mu_r \left( \frac{T_r + 198.6}{T + 198.6} \right) \left( \frac{T_1}{T_r} \right)^{3/2} \quad (22)$$

where

$$\mu_r = 3.8 \times 10^{-7} \text{ slugs/ft-sec}$$

$$T_r = 524.4^\circ\text{R}$$

$$T_i = (T + 460)^\circ\text{R}$$

$$T = ^\circ\text{F}$$

In determining the qualifying conditions, the initial pressure,  $P_i$ , is used in Eqs. (14) through (19), resulting in worst case solutions.

## 5.2 RAMP PROGRAM

Although Eq. (6) is an exact solution and could be programmed despite the relative complexity of the Weber functions, it was apparent that numerical integration of the differential equation, Eq. (II-27), is more practical. Since the initial value of the transducer pressure is known, the Runge-Kutta method, a method for stepwise solution of initial-value problems, is employed. The accuracy of the Runge-Kutta method is dependent upon the magnitude of the increments of the independent variable,  $t$ . The program is set up so that the initial value of transducer pressure,  $P_o$ , may differ from the initial value of orifice pressure  $P_i$ . Hence, if  $P_f = P_i$  (zero slope) and  $P_o \neq P_i$ , then the ramp is actually a step with  $\delta = P_i - P_o$ . To determine the size of the integration increments, the same step and system were applied to both programs and the increment adjusted until the solutions coalesced, as shown in Fig. 7.

The ramp program is currently written in Fortran II for an IBM 7074. The standard input sheet is shown in Fig. 8. Since the initial transducer pressure,  $P_o$ , can be specified independently of the orifice pressure, solutions can be combined by using the transducer pressure found at the end of the first ramp as the initial transducer pressure for the second ramp and so on. In this manner, complex orifice pressure-time relationships can be approximately synthesized using a series of ramp functions and the transducer response determined.

Although 5000 integrations are performed per period  $\tau$ , only the results of integration at every  $1/10 \tau$  are printed out. A typical output appears in Fig. 9.

In determining the qualifying conditions,  $P = P_o$  and  $\delta = P_f - P_o$  are used.

### 5.3 OTHER PROGRAMS

The techniques employed for the ramp forcing function may be applied in like manner for other functions. One method is, of course, to synthesize the function with a series of ramps as discussed previously. Or, the Riccati equation, Eq. (II-27), with the corresponding forcing function  $f(t)$  found by using Eq. (II-28), may be integrated numerically. For example, if the orifice pressure consisted of a sinusoid  $B \sin \omega t$  superimposed upon a steady level  $A$ , the forcing function would be

$$f(t) = \frac{1}{K_m} \left\{ (A + B \sin \omega t)^2 - K_T \omega B \cos \omega t \right\} \quad (23)$$

## SECTION VI

### DISCUSSION OF PROGRAM UTILITY

The selection of the better one of the two programs to use for a particular problem depends upon what forcing function is to be used and what information is required. Each program has certain unique features. The step program, for example, is the best program to use when comparing systems because of its Option 5, compute  $K^*$  only. In addition, it must be used for system optimization. The ramp program achieves its greatest utility when used to synthesize complex forcing functions. Both programs can study system response to a step forcing function but have different outputs. The step program is preferred when  $t_L$  is to be determined, and the ramp program is preferred when a plot of the response is desired. The remainder of this section is devoted to the discussion of typical uses of the programs, interpretation of data, and presentation of results.

### 6.1 DIAMETER OPTIMIZATION

The theoretical lag time equations have been shown (Refs. 1 and 2) to predict measured lag times to within 10 percent for pressures greater than 50 psf; however, in the optimization of a two-tube system the most important consideration is not the accuracy of the lag time but rather the accuracy of the determination of the diameter of the second tube which will minimize the lag time. Since the optimization equations were not verified in either Refs. 1 or 2, a laboratory setup was employed to experimentally verify optimization. The results of this work, given in Fig. 10, show the expected error in  $K^*$  values but only negligible error in the value of the optimum diameter. Figure 11 shows some of the measured time lag data obtained for one of the five tubing configurations used in the experimental verification.

A plot of the optimum diameters and  $K^*$  values for three different orifice diameters is shown in Fig. 12. These data were generated by the step program to determine orifice size required to limit the time lag to a reasonable value and to determine the appropriate  $d_2$  of a three-tube system to be used during an actual test in Tunnel 16T. An interesting point is demonstrated in Fig. 12. As shown, for a tubing system in which the first and third tubes are fixed, the diameter of the second tube may be smaller or larger than the diameter of the third tube, depending upon the other tubing dimensions. This is contrary to the generally accepted intuitive deduction that purports that an optimum tubing system is always a stepwise, uniformly tapered system, i. e., each successive tube diameter is larger than the previous diameter as one goes from orifice to transducer.

Since the use of an optimum diameter tube would normally involve complete installation of new tubing for each test, which would be both costly and time consuming, the question arises as to whether one, or perhaps two, tube diameters might represent a reasonable compromise. The step program can be used to logically determine a best compromise as the following example study demonstrates.

In this example study, model orifice diameters of 0.020, 0.030, 0.040, 0.050, 0.060, and 0.070 in. were assumed representative of orifices normally encountered in a series of model test programs. Each diameter was given the same weighting, i. e., all six diameters were assumed to receive equal usage. The remainder of the two-tube measurement system used in the study is explained in Fig. 13. The three combinations of compromise diameters studied are explained in Table I.

**TABLE I**  
**EXPLANATION OF COMBINATIONS EMPLOYED IN COMPROMISE**  
**DIAMETER STUDY**

Combination	Compromise	Orifice Diameters Compromised, in.
1	A	0.020, 0.030, 0.040, 0.050, 0.060, 0.070
2	B	0.020, 0.030
	C	0.040, 0.050, 0.060, 0.070
3	D	0.020, 0.030, 0.040
	E	0.050, 0.060, 0.070

The deviation from the optimum case for a particular orifice diameter ( $d_1$ ) at a given diameter  $d_2$  is

$$\Delta K_i^* = K_i^* - (K_i^*)_{\text{opt}} \quad (24)$$

and the total deviation is given by

$$(\Delta K^*)_{\text{total}} = \sum_{i=20}^{70} \left\{ K_i^* - (K_i^*)_{\text{opt}} \right\} \quad (25)$$

where  $i = 20$  refers to  $d_1 = 0.020$  in. The compromise diameter for a specific compromise combination can be defined as the diameter that results in the minimum value of  $(\Delta K^*)_{\text{total}}$  given by

$$\left\{ (\Delta K^*)_{\text{total}} \right\}_{\min} = \sum_{i=20}^{70} \left\{ (K_i^*) - (K_i^*)_{\text{opt}} \right\}_{\min} \quad (26)$$

The most desirable combination would be one in which only one compromise diameter is used. This compromise, for the example study, was compromise A, shown in Fig. 13, with a compromise tube diameter ( $d_2$ ) of 0.058 in. and a total deviation of

$$\left\{ (\Delta K^*)_{\text{total}} \right\}_{\min} = 4.37$$

Referring to Fig. 13, it is apparent that the smaller the orifice diameter, the more important is the diameter of the second tube because of the steepness of the curve and larger magnitudes of  $K^*$ . Combination 2, compromised of compromises B and C, resulted in two compromise diameters - one to be used for the smaller orifices (0.020 and 0.030 in.), and the other for the larger orifices. These diameters, shown in Fig. 13, are 0.048 and 0.090 in., respectively. The total deviation resulting from the use of these two compromises was

$$\left\{ (\Delta K^*)_{\text{total}} \right\}_{\min} = 0.79$$

Combination 3 was studied to determine whether the 0.040-in. orifice should be grouped with the 0.020- and 0.030 in. orifices. The resulting diameters, compromises D and E in Fig. 13, yielded a deviation of

$$\left\{ (\Delta K^*)_{\text{total}} \right\}_{\min} = 1.40$$

Thus, combination 2 was the best compromise of the combination studied.

## 6.2 RAMP INPUT

Transducer responses to typical ramps of positive and negative slopes are shown in Figs. 14 through 16. In each case, the response was determined using both the computer and the approximate equation. As shown in Figs. 14 and 15, the approximate equation results are quite close to the computer results whenever the input and response slopes are nearly equal. This was to be expected since the approximate solution was based upon the assumption of equal slopes. The approximate response shown in Fig. 16 is indicative of the discrepancies that can occur whenever the approximation assumptions are not valid.

Examples of synthesizing complex inputs with ramp segments are shown in Figs. 17 through 19. The first (Fig. 17) is representative of a pressure pumpdown curve. The second input (Fig. 18) is typical of the pressure-time relationship resulting from traversing a nozzle exit at a constant rate. Figure 19 is an example of synthesizing a periodic waveform. In this example, the transducer pressure was initially set at the lowest pressure rather than the average value of the input pressure in order to demonstrate that the computer program would give both the transient and steady-state responses. In Fig. 19, the attenuation, phase shift, and transient are quite apparent.

## REFERENCES

1. Bauer, R. C. "A Method of Calculating the Response Time of Pressure Measuring Systems." AEDC-TR-56-7 (AD98978), November 1956.
2. Kinslow, Max. "Correction for Lag Time in Pressure Measuring Systems." AEDC-TR-58-8 (AD161040), August 1958.
3. Handbook of Mathematical Functions. Applied Mathematics Series 55, C13.32:55 NBS, U.S. Government Printing Office, Washington, D. C.
4. Murphy, George M. Ordinary Differential Equations and Their Solutions. Van Nostrand Company, Inc., Princeton, New Jersey, 1960.



**APPENDIXES**

- I. ILLUSTRATIONS**
- II. DERIVATION OF SYSTEM EQUATIONS**
- III. STEP FORCING FUNCTION EQUATION LIST**
- IV. RAMP FORCING FUNCTION EQUATION LIST**



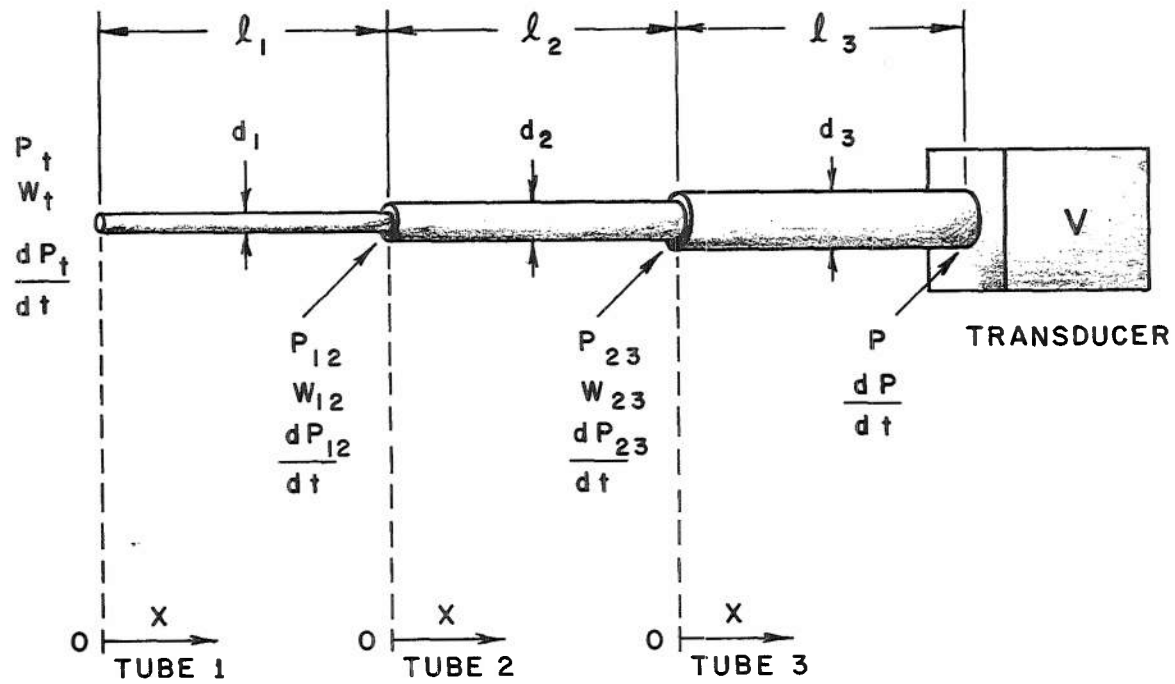


Fig. 1 Three-Tube Pressure Measurement System

PWT  
7/12/65INPUT INSTRUCTIONSTHREE-TUBE PRESSURE LAG COMPUTER PROGRAMStep Forcing Function

<u>Item</u>	<u>Instructions</u>			
1	yXXXX	where:	y = 1 for letter S y = 2 " " T y = 3 " " A y = 4 " " B y = 5 " " L	
2	Numeric			
4	Omit "19"			
5	Option Code	1	Error = psf	$\Delta p$ = psf
		2	" - % Final Val.	" = psf
		3	" = psf	" = % Initial Val.
		4	" = % Final Val.	" = % Initial Val.
		5	K* Computation Only	
		6	d <sub>2</sub> Optimum & K* Minimum	
8	FT <sup>3</sup>			
9-14	FT			
10&13	Zero for 1-Tube			
11&14	Zero for 2-Tube			
13	Zero for Option 6			
15	Unity for Options 5 & 6, $\leq 16$			
16	Zero " " " "			
17	Unity " " " " $\leq 28$			
18	Zero " " " "			

Fig. 2 Standard Input Sheet for Step Program

# INPUTS

## THREE - TUBE PRESSURE LAG COMPUTER PROGRAM STEP FORCING FUNCTION

ITEM

CONTACT \_\_\_\_\_  
SHEET \_\_\_\_ OF \_\_\_\_

1. Project							
2. Month							
3. Day							
4. Year							
5. Option No.							
6. Run No.							
7. Error							
8. Xducer Volume							
9. Length 1							
10. Length 2							
11. Length 3							
12. Diameter 1							
13. Diameter 2							
14. Diameter 3							
15. $n\Delta p$							
16. n values of $\Delta p$ ↓							
17. $n P_i$							
18. n values of $P_i$ ↓							

Fig. 2 Concluded

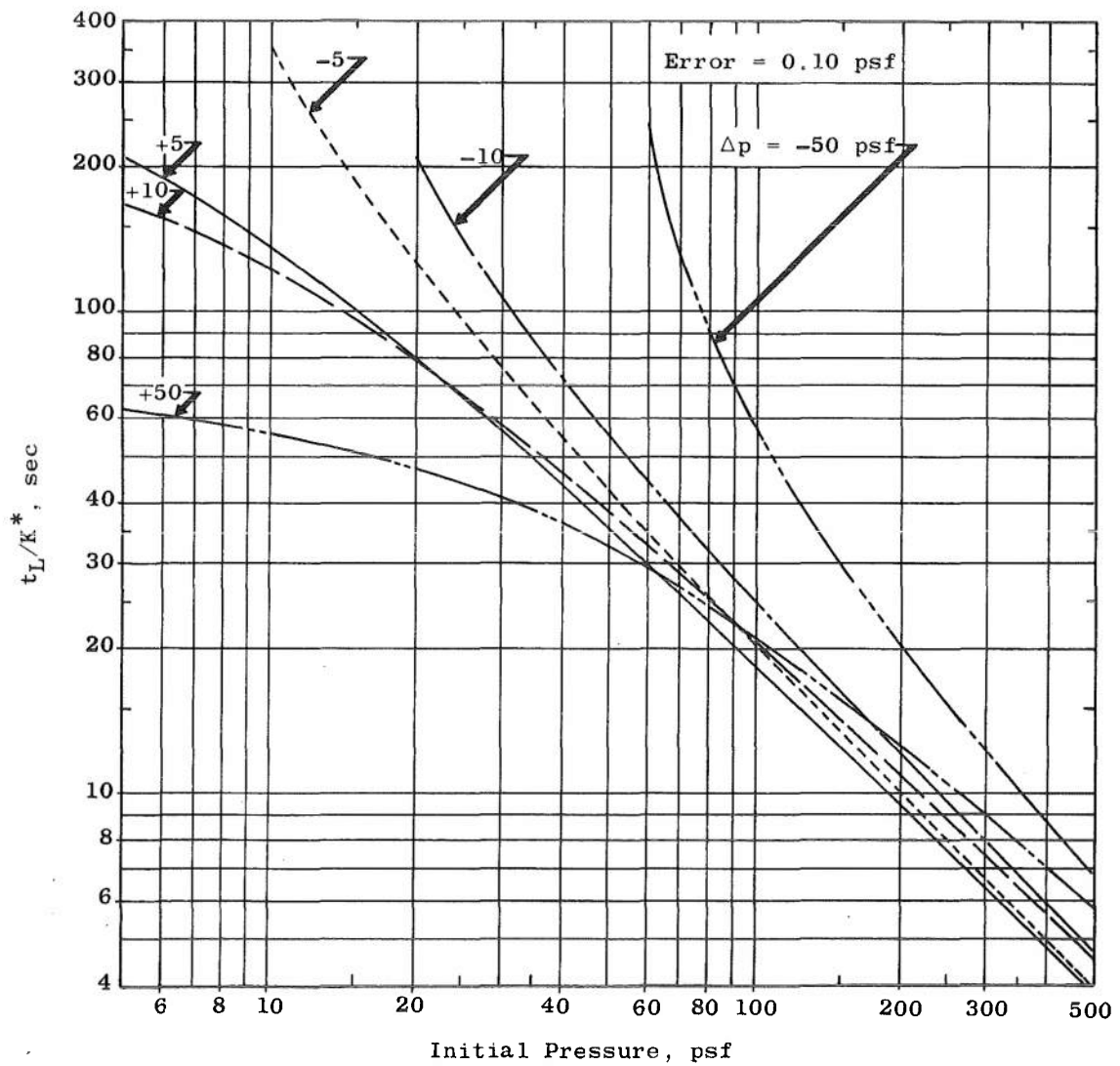


Fig. 3 Generalized Plots of Program Calculated Pressure Lag Times – Step Forcing Function

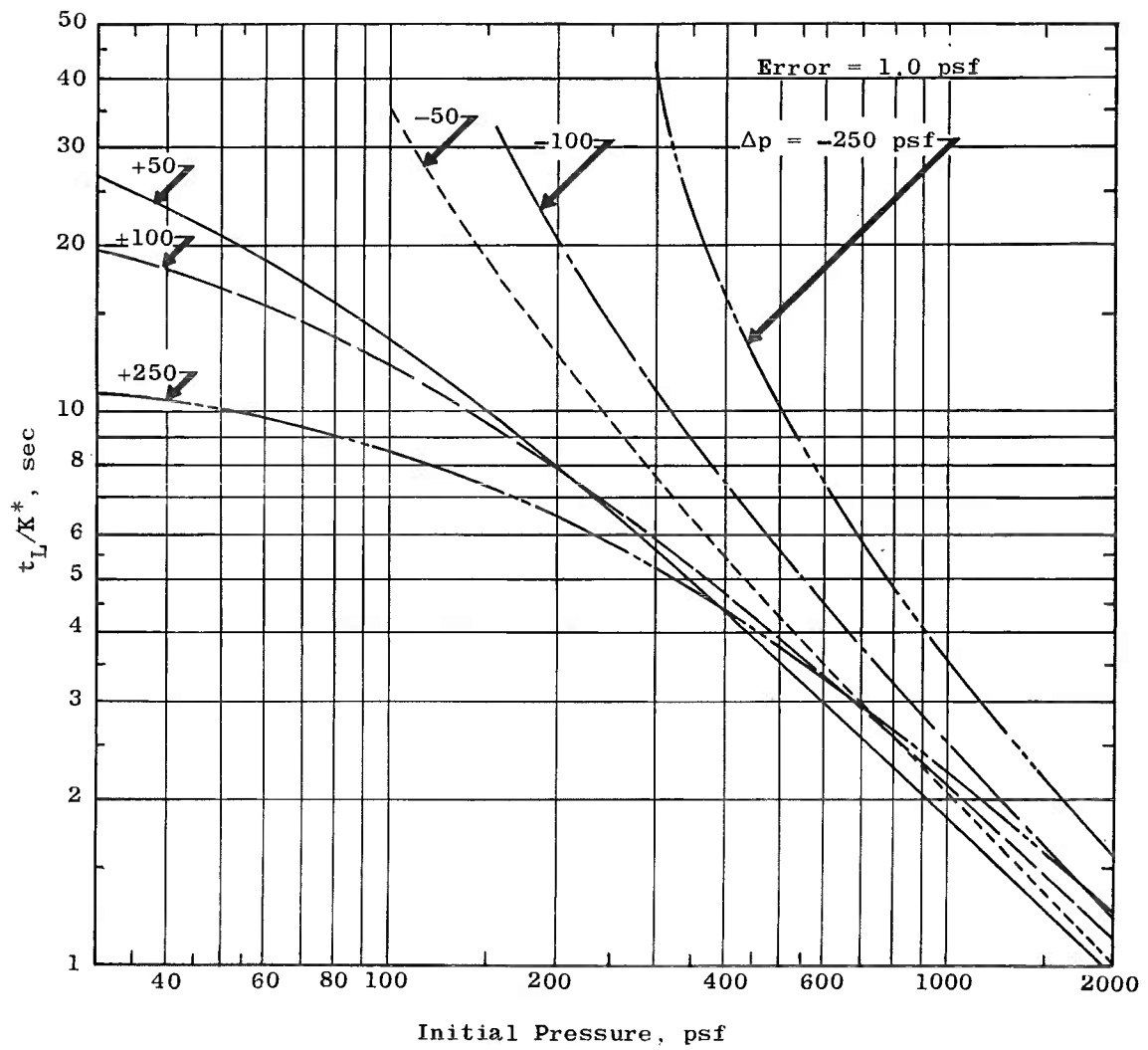


Fig. 3 Continued

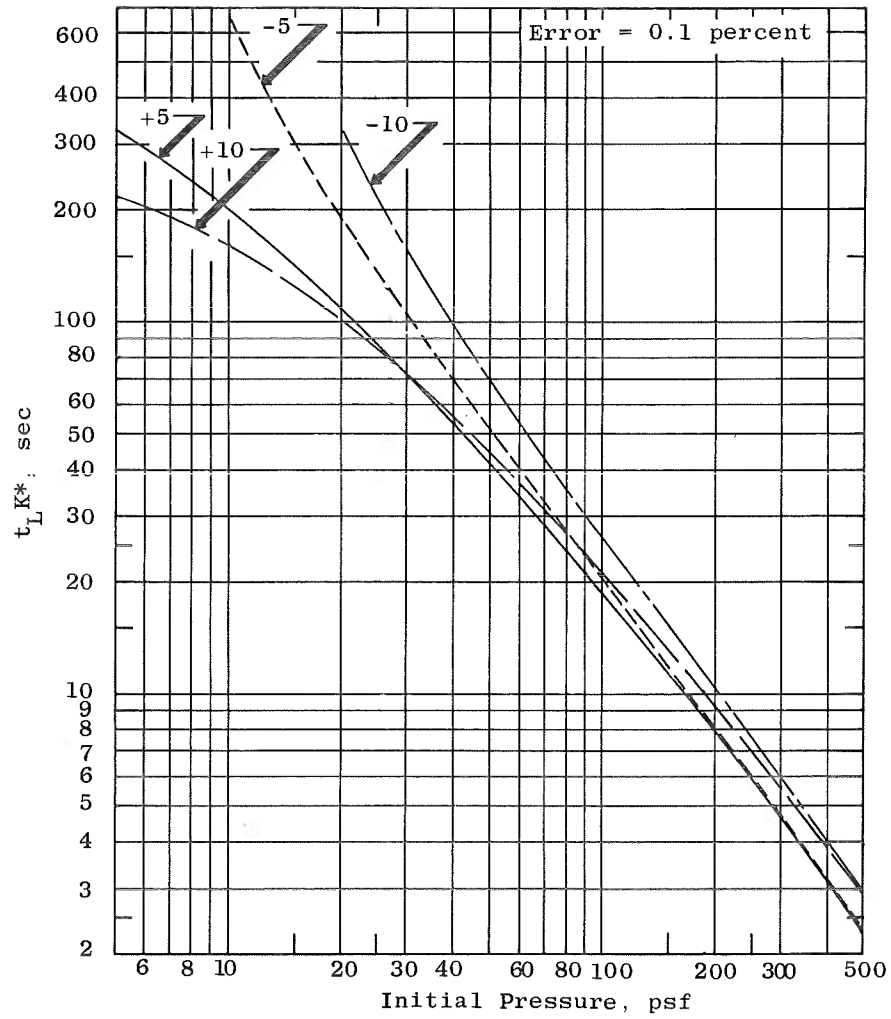


Fig. 3 Continued



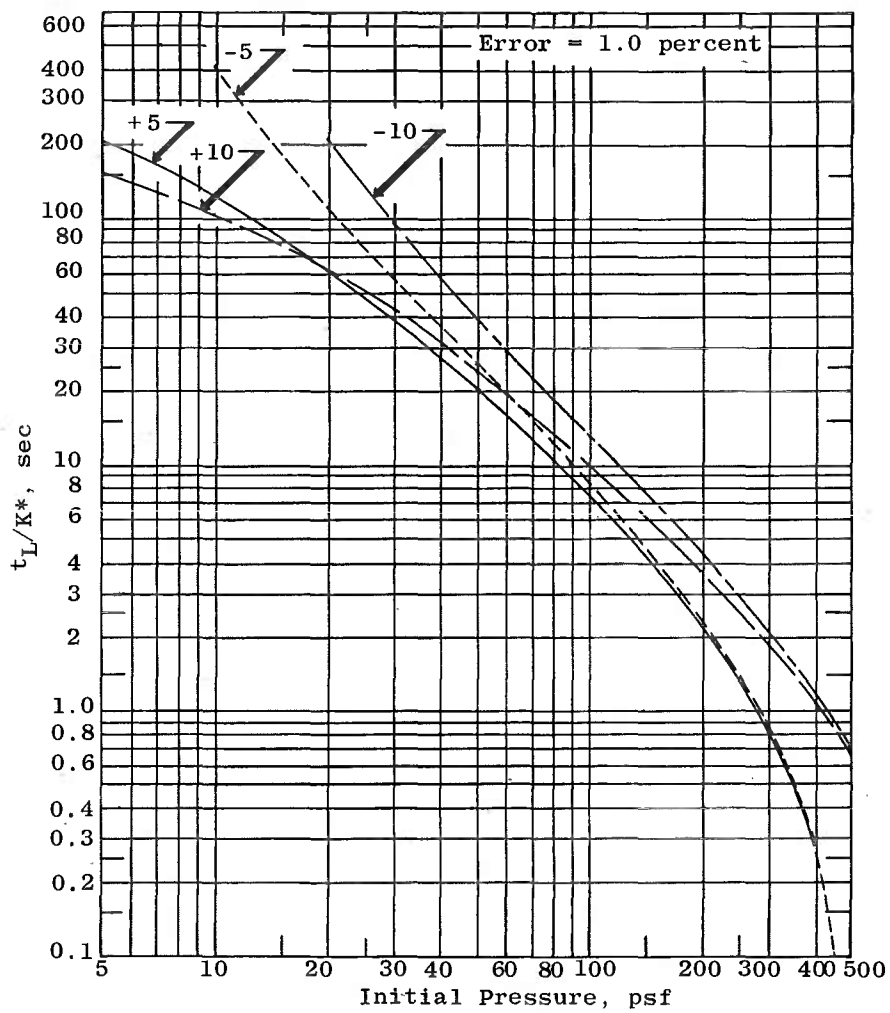


Fig. 3 Continued

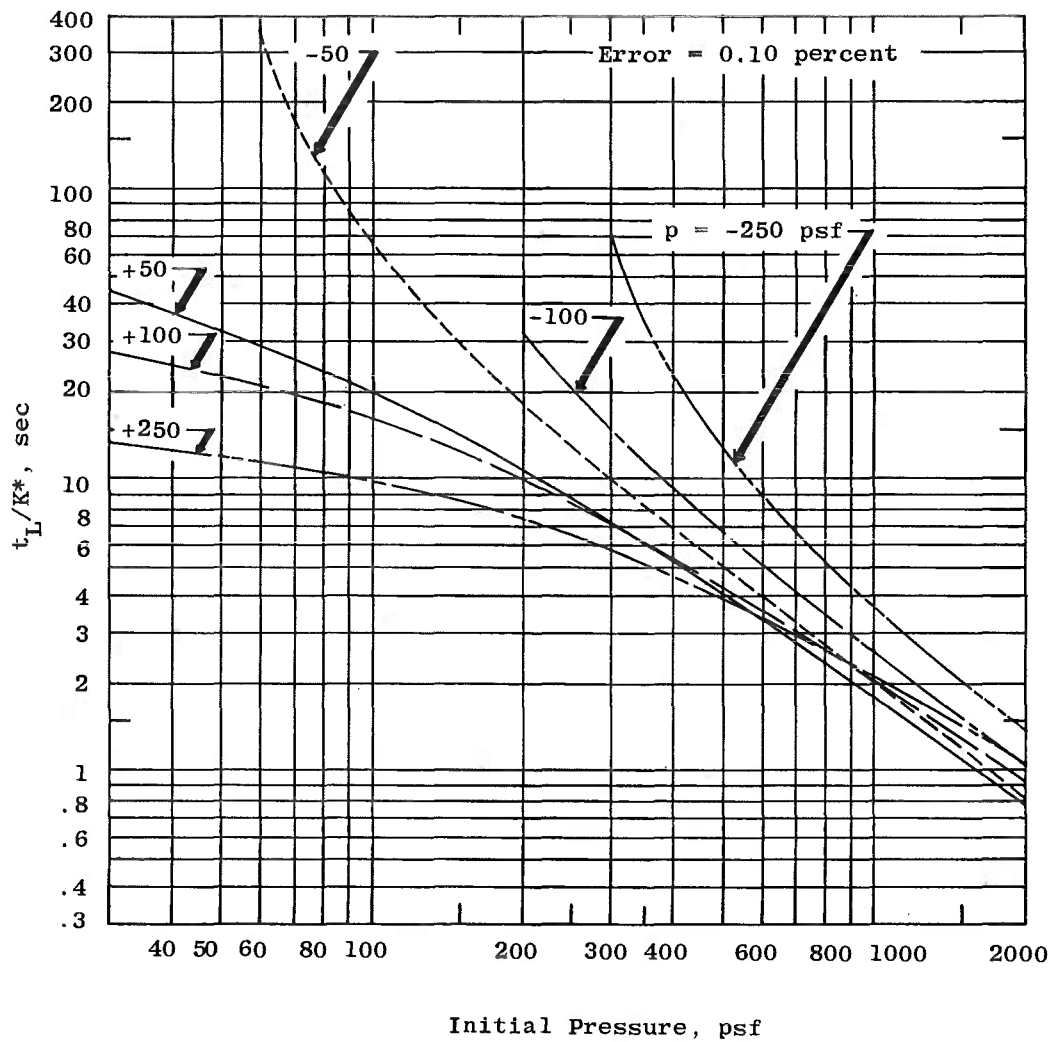


Fig. 3 Continued

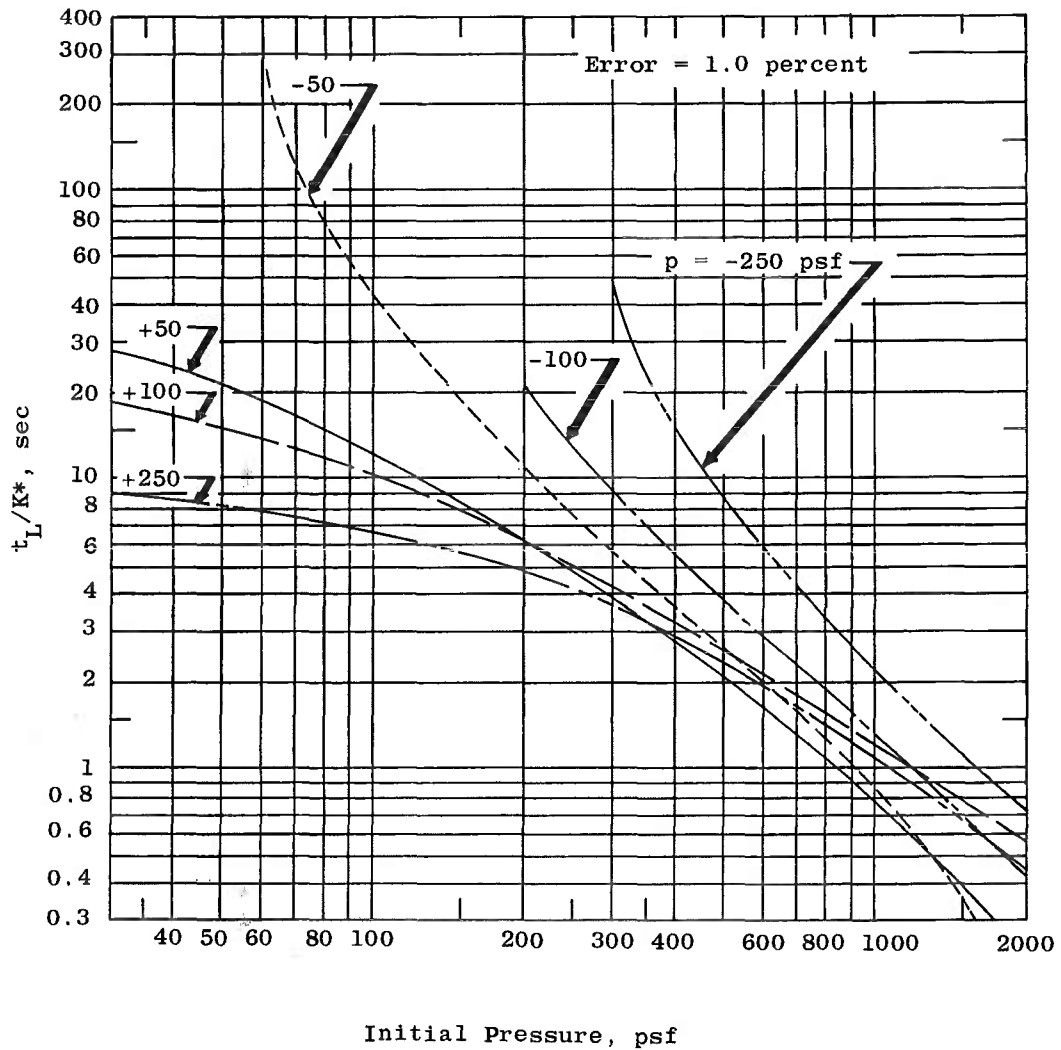


Fig. 3 Continued

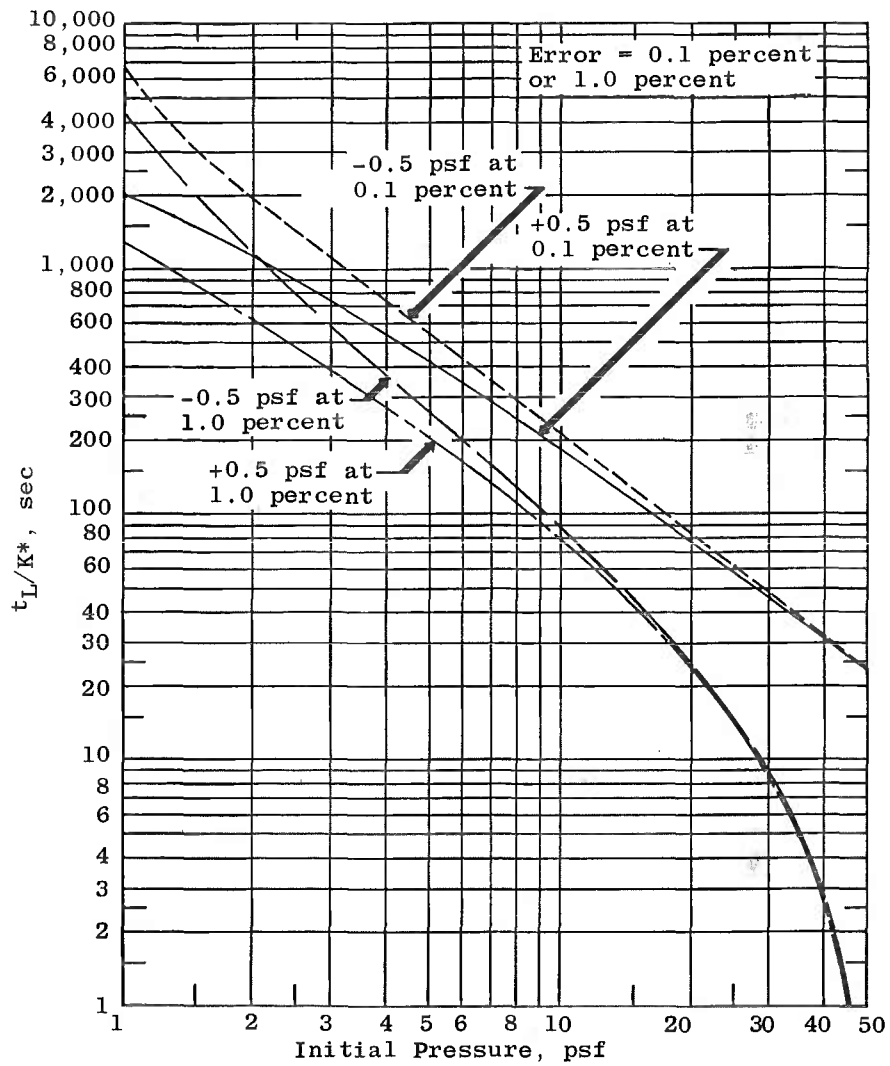


Fig. 3 Concluded

3 - Tube Pressure Lag 10-18-65 PT0617

Run No.	Option	K*	€	δ	V	ℓ 1	ℓ 2	ℓ 3	d1	d2	d3
5.0000+00	1.0000+00	9.1450+00	1.0000+00	1.0000+02	3.5000-04	1.5000+01	8.5000+01	3.4000+01	3.9160-03	7.5000-03	5.4170-03

P1	tL	Re 1	Re 2	Re 3	a1	a2	a3
2.0000+03	1.0079+01	9.6973+01	4.9781+01	6.8063+01	4.0141+04	5.4013+05	1.4699+05
1.8000+03	1.1146+01	8.7628+01	4.4903+01	6.1308+01	4.0141+04	5.4010+05	1.4699+05
1.6000+03	1.2466+01	7.8283+01	4.0024+01	5.4553+01	4.0140+04	5.4011+05	1.4699+05
1.4000+03	1.4140+01	6.8939+01	3.5145+01	4.7798+01	4.0139+04	5.4013+05	1.4698+05
1.2000+03	1.6334+01	5.9594+01	3.0266+01	4.1043+01	4.0138+04	5.4012+05	1.4698+05
1.0000+03	1.9334+01	5.0250+01	2.5387+01	3.4288+01	4.0136+04	5.4012+05	1.4698+05
8.0000+02	2.3684+01	4.0905+01	2.0508+01	2.7533+01	4.0132+04	5.4009+05	1.4697+05
6.0000+02	3.0561+01	3.1561+01	1.5629+01	2.0778+01	4.0125+04	5.4008+05	1.4696+05
4.0000+02	4.3069+01	2.2216+01	1.0750+01	1.4023+01	4.0108+04	5.4002+05	1.4692+05
2.0000+02	7.2944+01	1.2872+01	5.8710+00	7.2675+00	4.0037+04	5.3979+05	1.4674+05

Fig. 4 Typical Step Program Output - Option 1

3 - Tube Pressure Lag 10-18-65 PT0617

Run No.	Option	K*	V	$\ell 1$	$\ell 2$	$\ell 3$	d1	d2	d3
1.0000+01	5.0000+00	9.1450+00	3.5000-04	1.5000+01	8.5000+01	3.4000+01	3.9160-03	7.5000-03	5.4170-03
2.0000+01	5.0000+00	9.3850+00	3.5000-04	1.5000+01	8.5000+01	4.0000+01	3.9160-03	7.5000-03	5.4170-03
3.0000+01	5.0000+00	9.7149+00	3.5000-04	1.6000+01	8.5000+01	3.4000+01	3.9160-03	7.5000-03	5.4170-03
4.0000+01	5.0000+00	2.9790+00	1.3500-03	1.5000+01	1.0000+00	0.0000+00	3.9160-03	3.9160-03	7.5000-03
5.0000+01	5.0000+00	1.1458+01	1.3500-03	1.5000+01	8.5000+01	1.0000+00	3.9160-03	7.5000-03	3.9160-03

Fig. 5 Typical Step Program Output - Option 5

3 - Tube Pressure Lag 10-18-65 PT0617

Run No.	Option	(d2)opt	K*min	K*-6	K*-3	K*+3	K*+6	V	L1	L2	L3	d1	d3
1.1000+01	6.0000+00	6.7765-03	8.9689+00	9.1276+00	9.0127+00	8.9843+00	9.0499+00	3.5000-04	1.5000+01	8.5000+01	3.4000+01	3.9160-03	5.4170-03
2.1000+01	6.0000+00	7.0949-03	9.2951+00	9.3729+00	9.3040+00	9.3369+00	9.4220+00	3.5000-04	1.5000+01	8.5000+01	4.0000+01	3.9160-03	5.4170-03
3.1000+01	6.0000+00	6.6880-03	9.4573+00	9.6177+00	9.4991+00	9.4795+00	9.5558+00	3.5000-04	1.6000+01	8.5000+01	3.4000+01	3.9160-03	5.4170-03
5.1000+01	6.0000+00	6.5955-03	1.0883+01	1.1009+01	1.0895+01	1.0956+01	1.1100+01	1.3500-03	1.5000+01	8.5000+01	1.0000+00	3.9160-03	3.9160-03

Fig. 6 Typical Step Program Output - Option 6

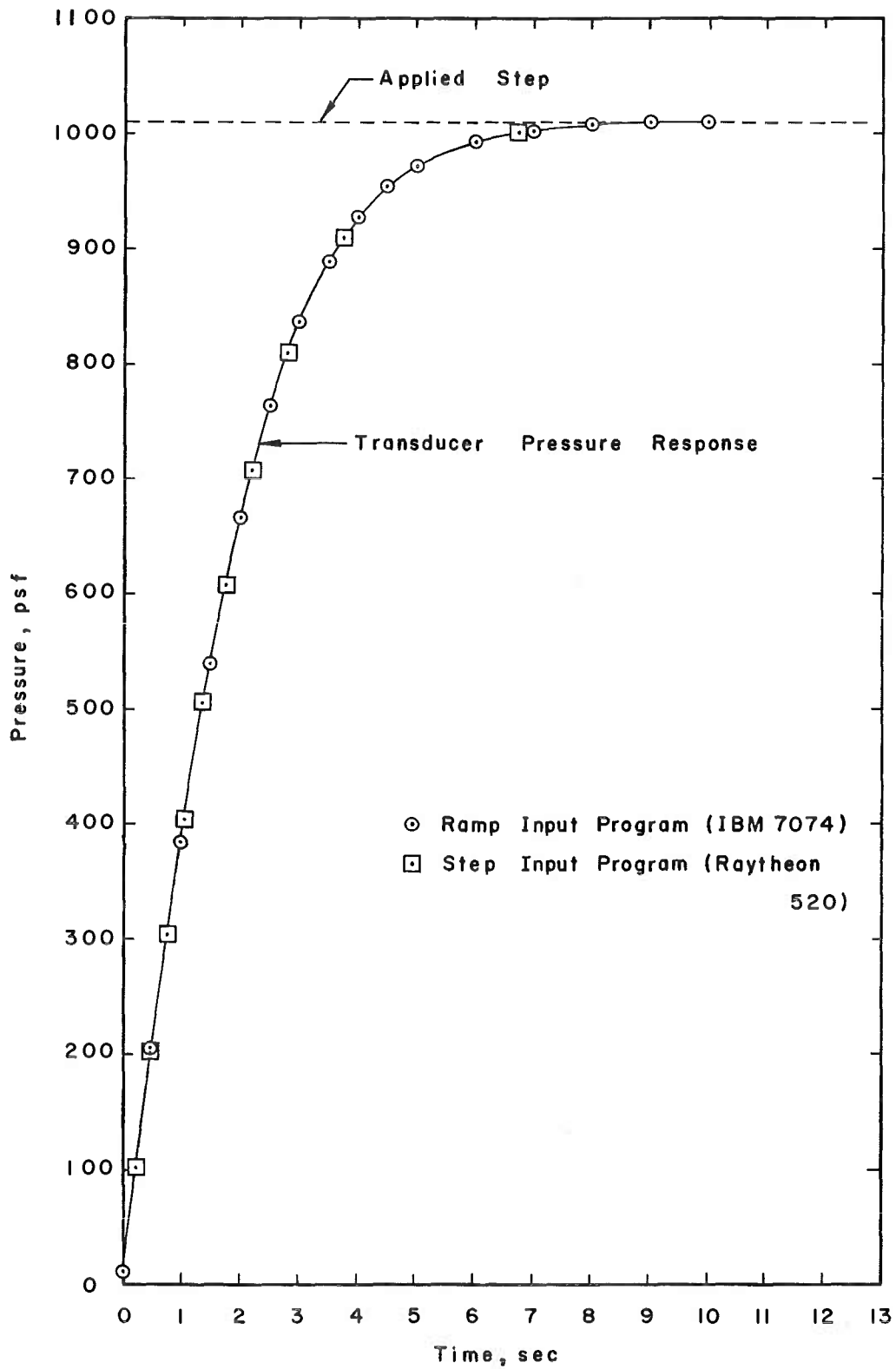


Fig. 7 Example of Step and Ramp Program Agreement



# INPUTS

## THREE - TUBE PRESSURE LAG COMPUTER PROGRAM RAMP FORCING FUNCTION

ITEM

CONTACT \_\_\_\_\_  
SHEET \_\_\_\_ OF \_\_\_\_

1. Project No.						
2. Month						
3. Day						
4. Year						
5. Run No.						
6. Xducer Volume						
7. Length 1						
8. Length 2						
9. Length 3						
10. Diameter 1						
11. Diameter 2						
12. Diameter 3						
13. Temperature						
14. Initial Orifice P.						
15. Final Orifice P.						
16. Initial Xducer P.						
17. Time Interval						

ItemInstructions

1	Example: PT8002
2	Numeric
4	Omit "19"
6	ft <sup>3</sup>
7-12	ft
8&11	Zero for 1-tube
9&12	Zero for 2-tube
13	°F
14-16	psf
17	sec

Fig. 8 Standard Input Sheet for Ramp Program

PROJ. NO. = PT -8002    DATE = 6-16-65    RUN NO. = 11

$\ell 1$	$\ell 2$	$\ell 3$	d1	d2	d3	v
2.0000+00	3.5000+01	3.3000+01	2.9200-03	8.1700-03	5.4200-03	3.5000-04
Km	KT	K*	T			
2.5856+03	1.3336+03	2.5856+00	7.2000+01			
Re1	Re2	Re3	a1	a2	a3	
3.9194+01	6.2276+00	7.5186+00	2.0274+03	2.4322+05	4.6452+04	
Po	Pi	Pf	T			
1.0000+02	1.0000+02	2.0000+02	1.0000+01			

TIME	P
1.0000+00	9.5425+01
2.0000+00	9.1997+01
3.0000+00	8.9702+01
4.0000+00	8.8533+01
5.0000+00	8.8488+01
6.0000+00	8.9569+01
7.0000+00	9.1773+01
8.0000+00	9.5095+01
9.0000+00	9.9524+01
1.0000+01	1.0504+02

Fig. 9 Typical Ramp Program Output

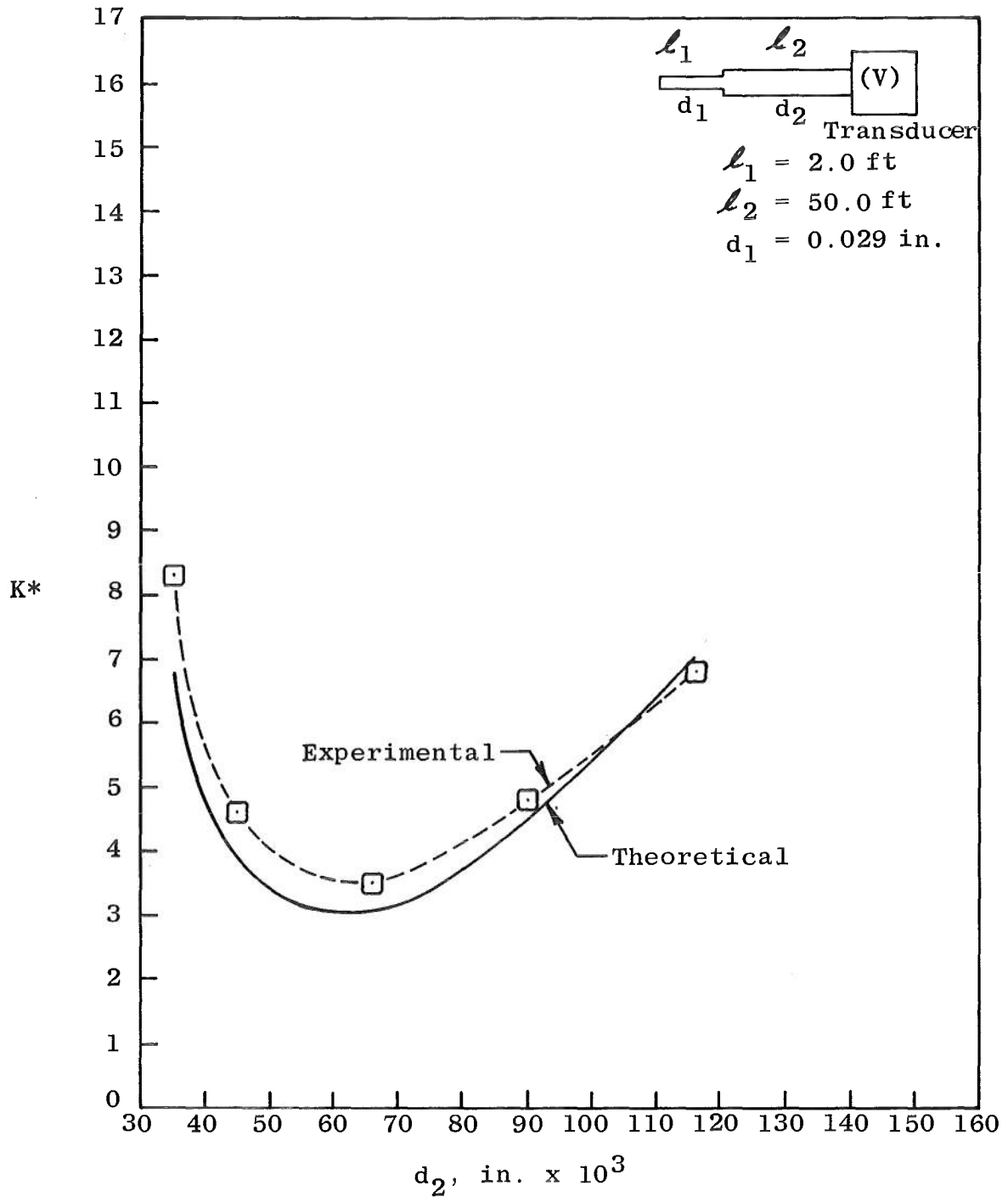


Fig. 10 Comparison of Experimental and Theoretical Values of  $K^*$  Showing Optimization

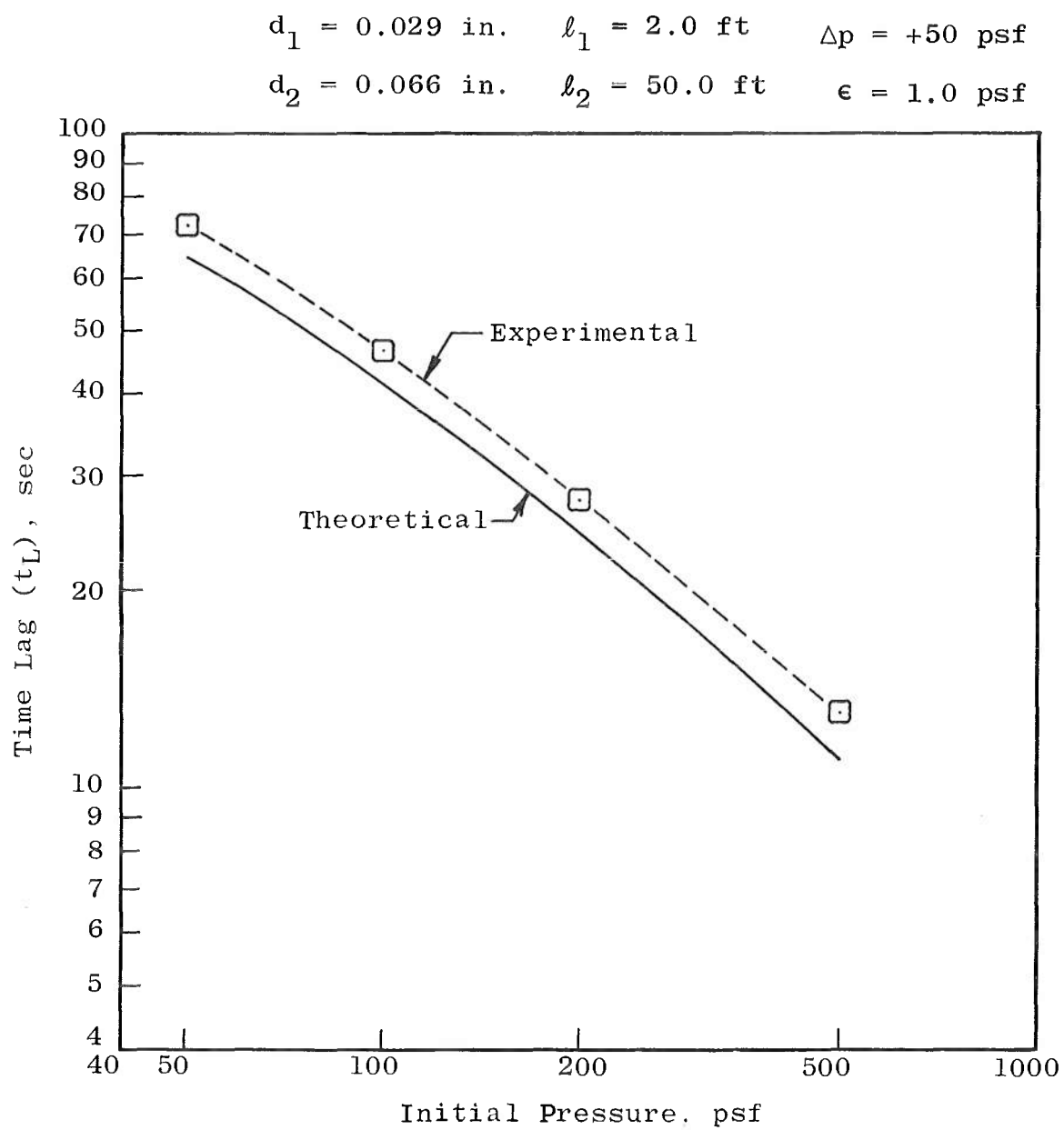


Fig. 11 Comparison of Experimental and Theoretical Time Lags Used in Diameter Optimization

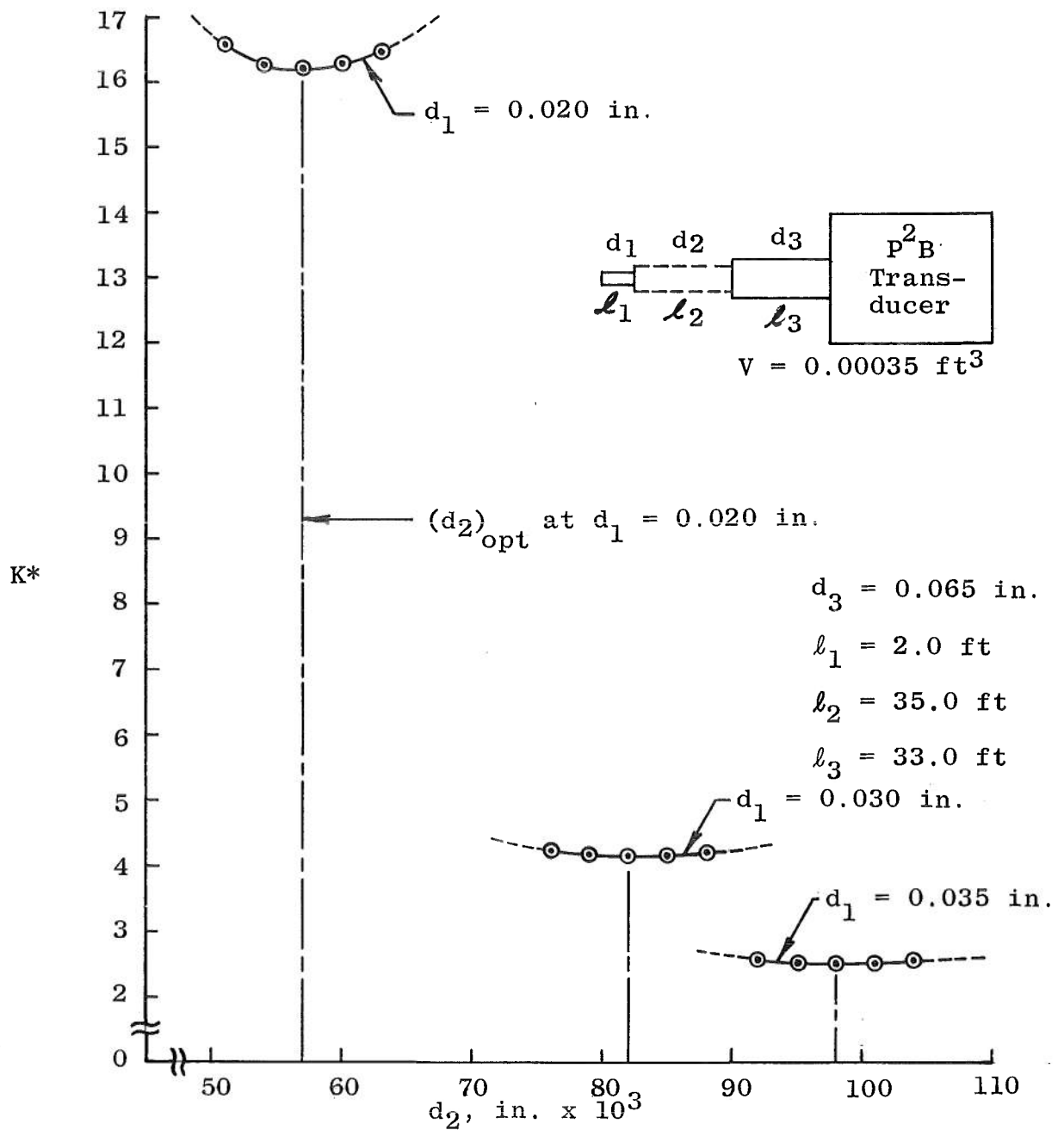


Fig. 12 Pressure Tubing Optimization for Actual Test Conducted in Tunnel 16T

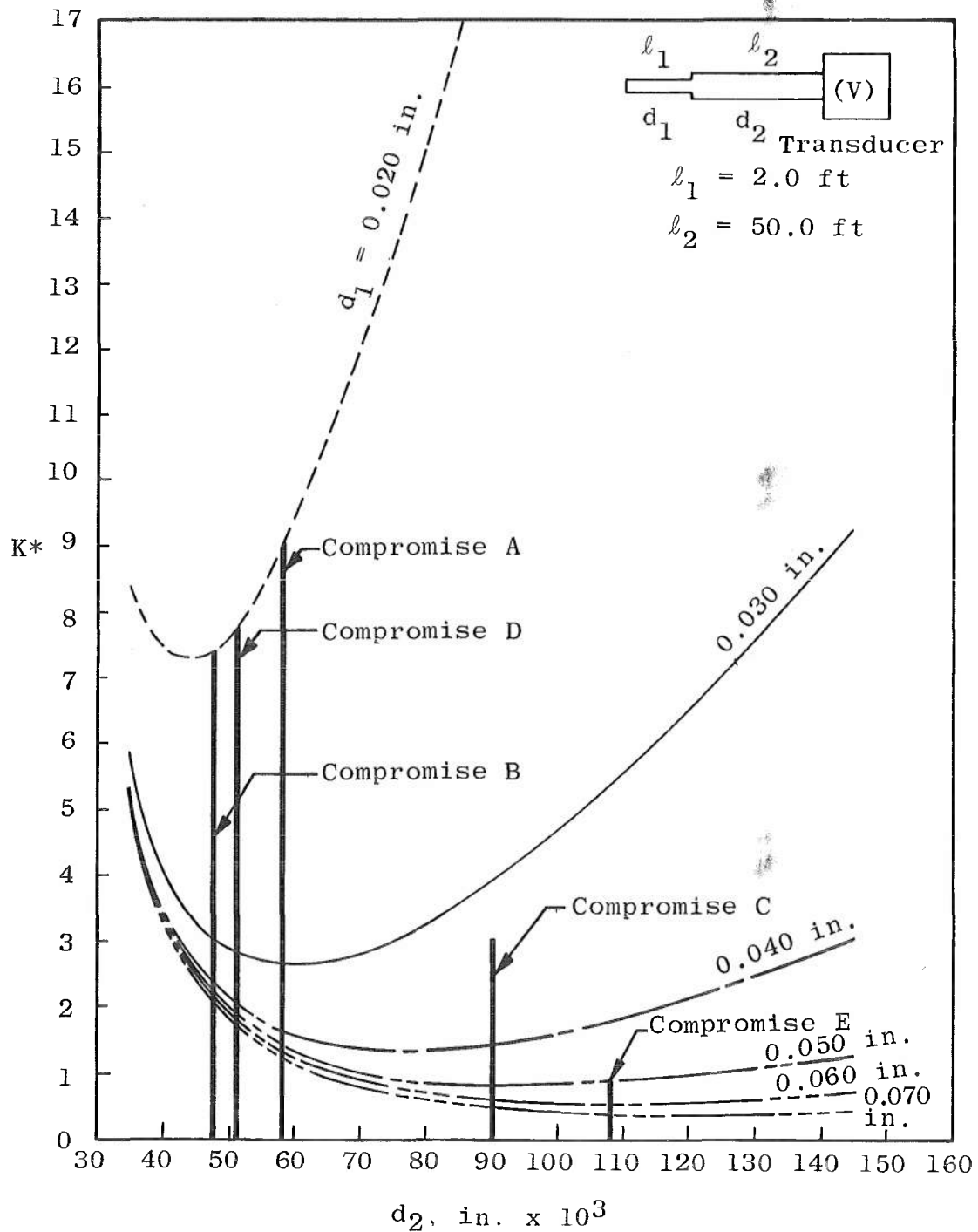


Fig. 13 Values of Geometric Characteristic  $K^*$  as Function of Input Orifice Diameter ( $d_1$ ) and Tube Diameter ( $d_2$ )

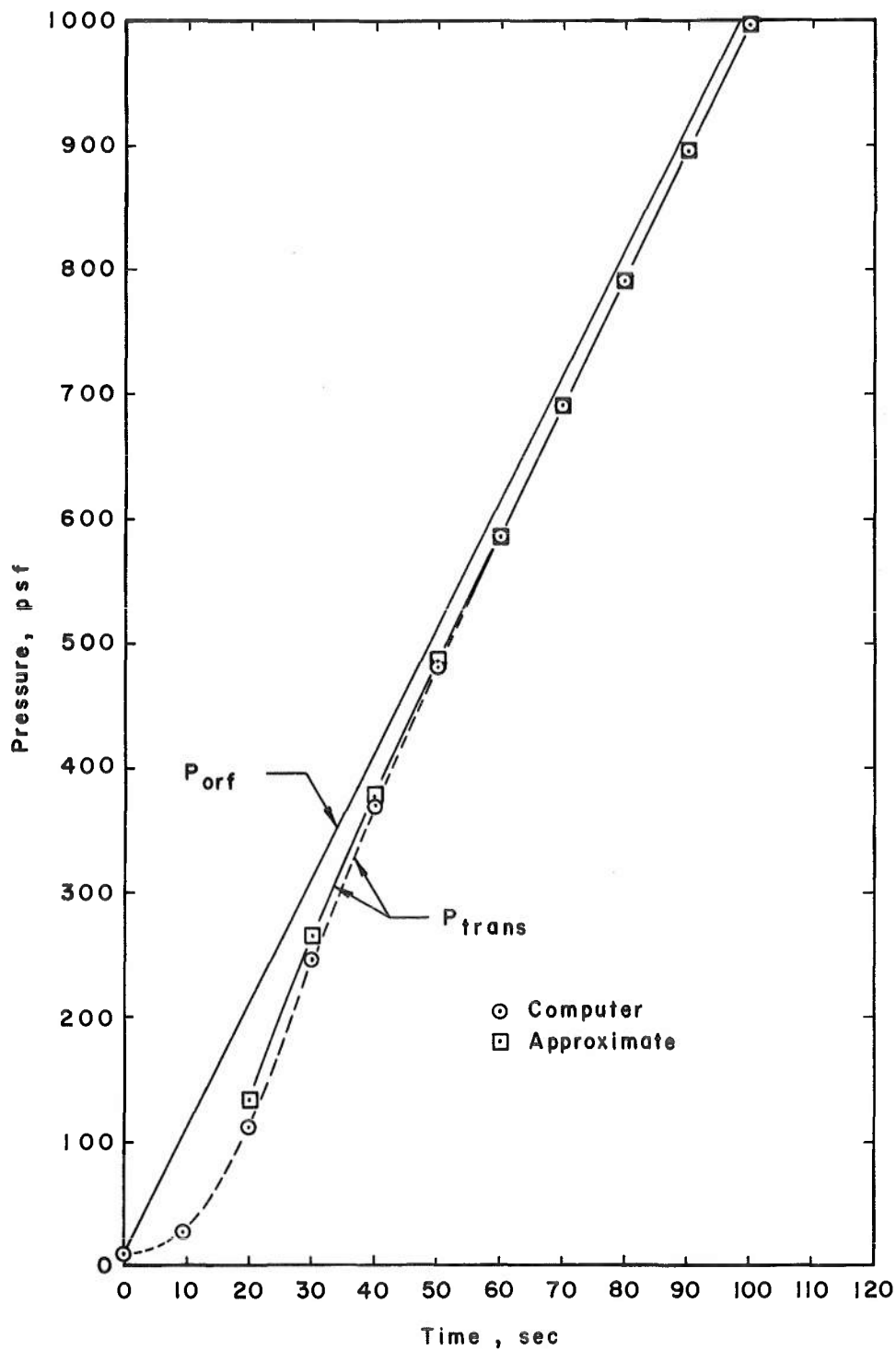


Fig. 14 Response to Ramp with Positive Slope

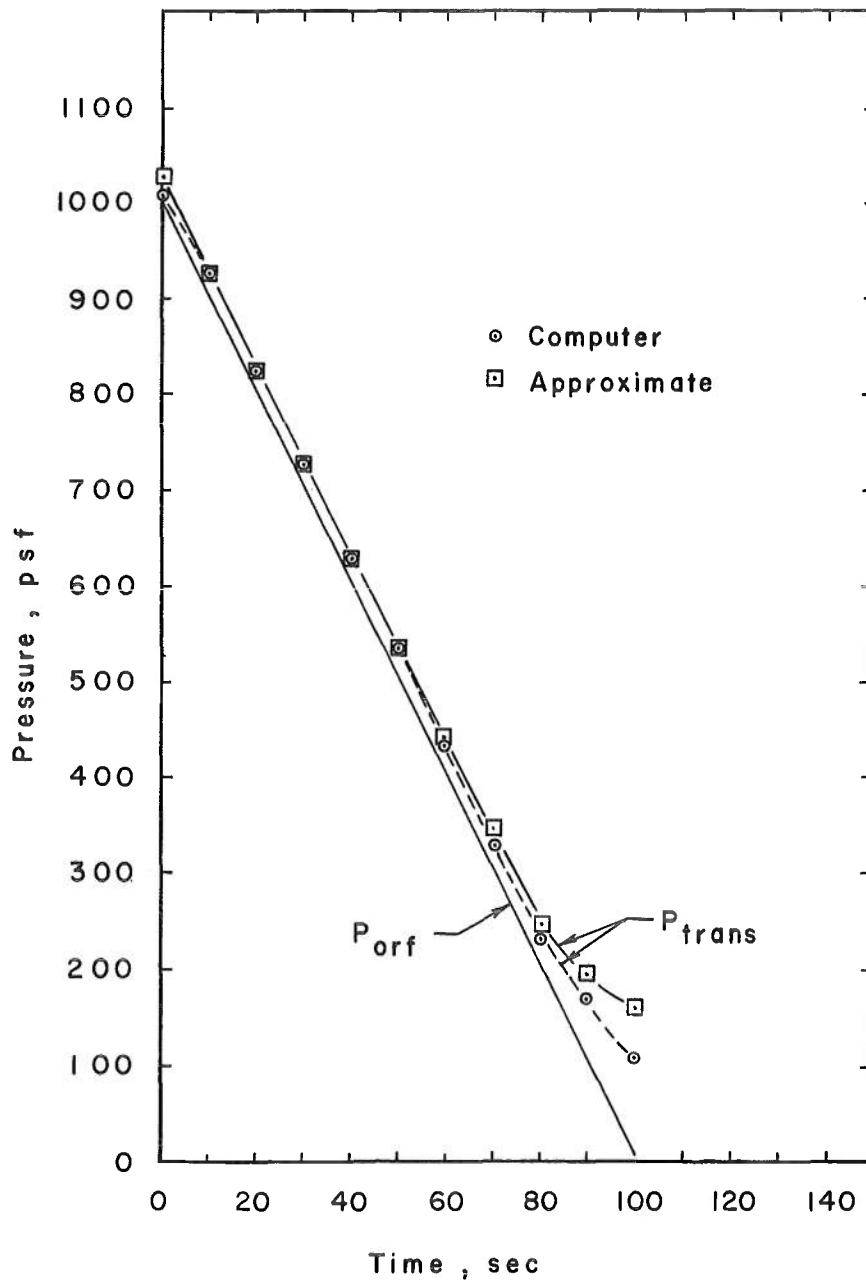


Fig. 15 Response to Ramp with Negative Slope



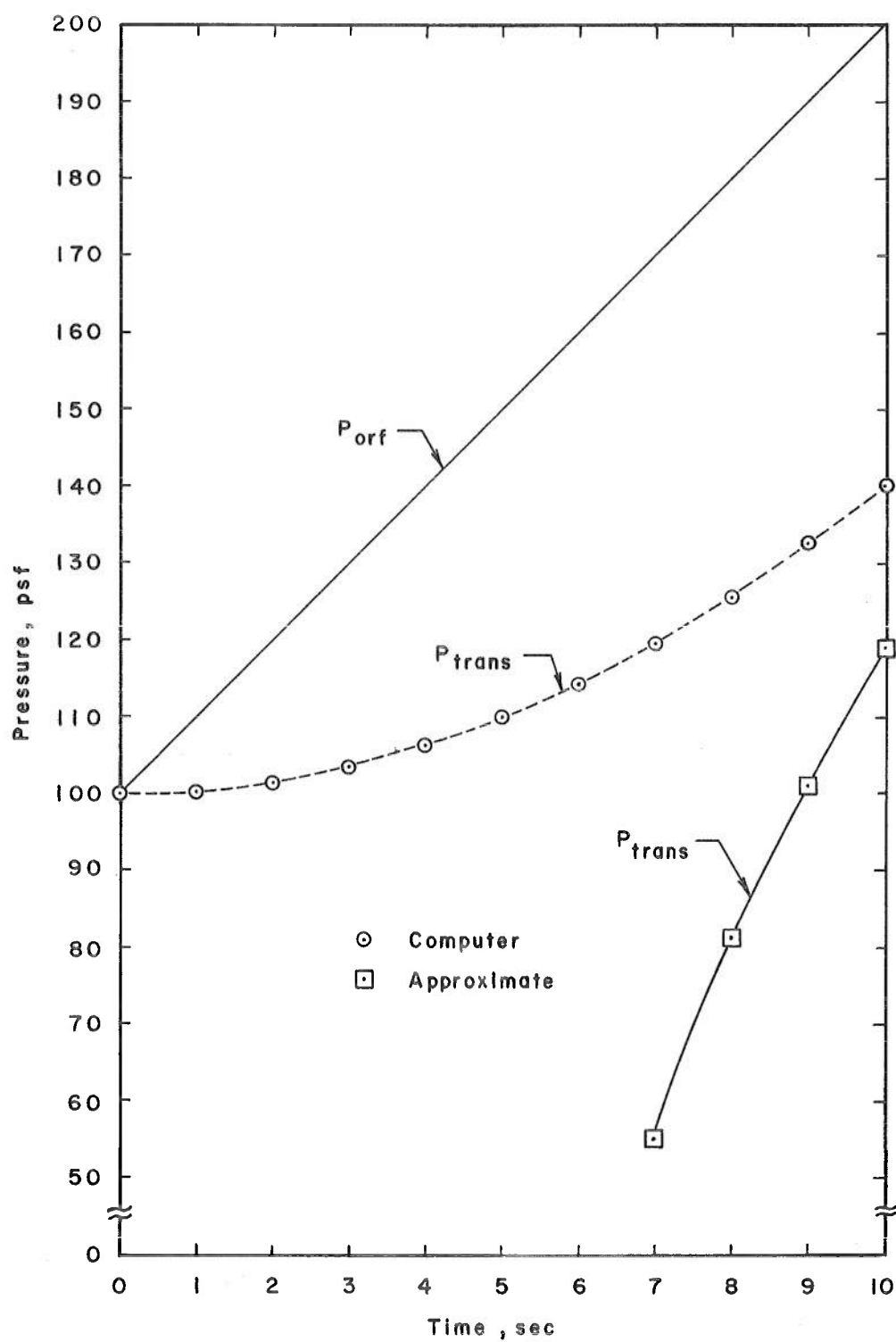


Fig. 16 Example in which the Approximate Ramp Solution Is Not Applicable

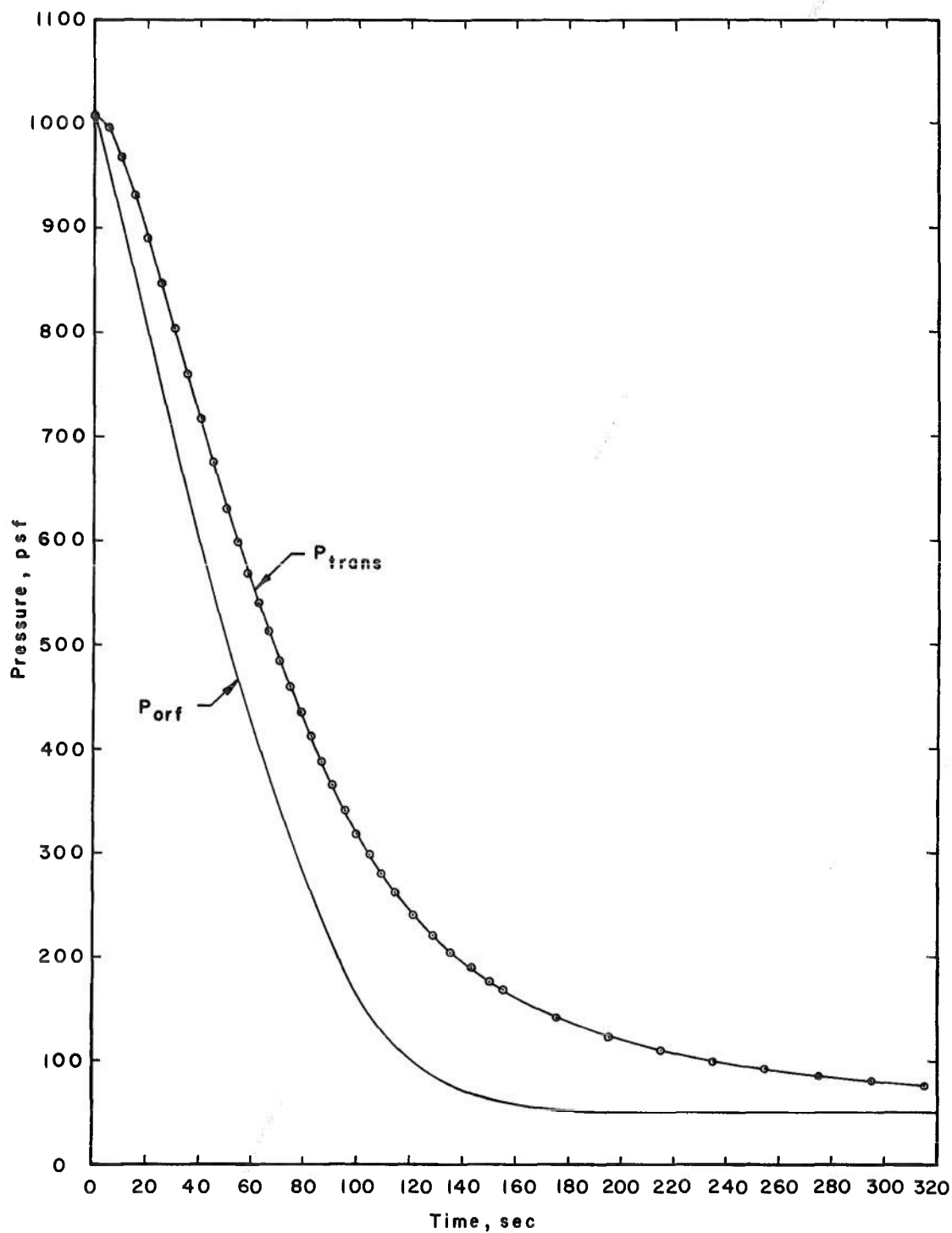


Fig. 17 Response to a Synthesized Pumpdown

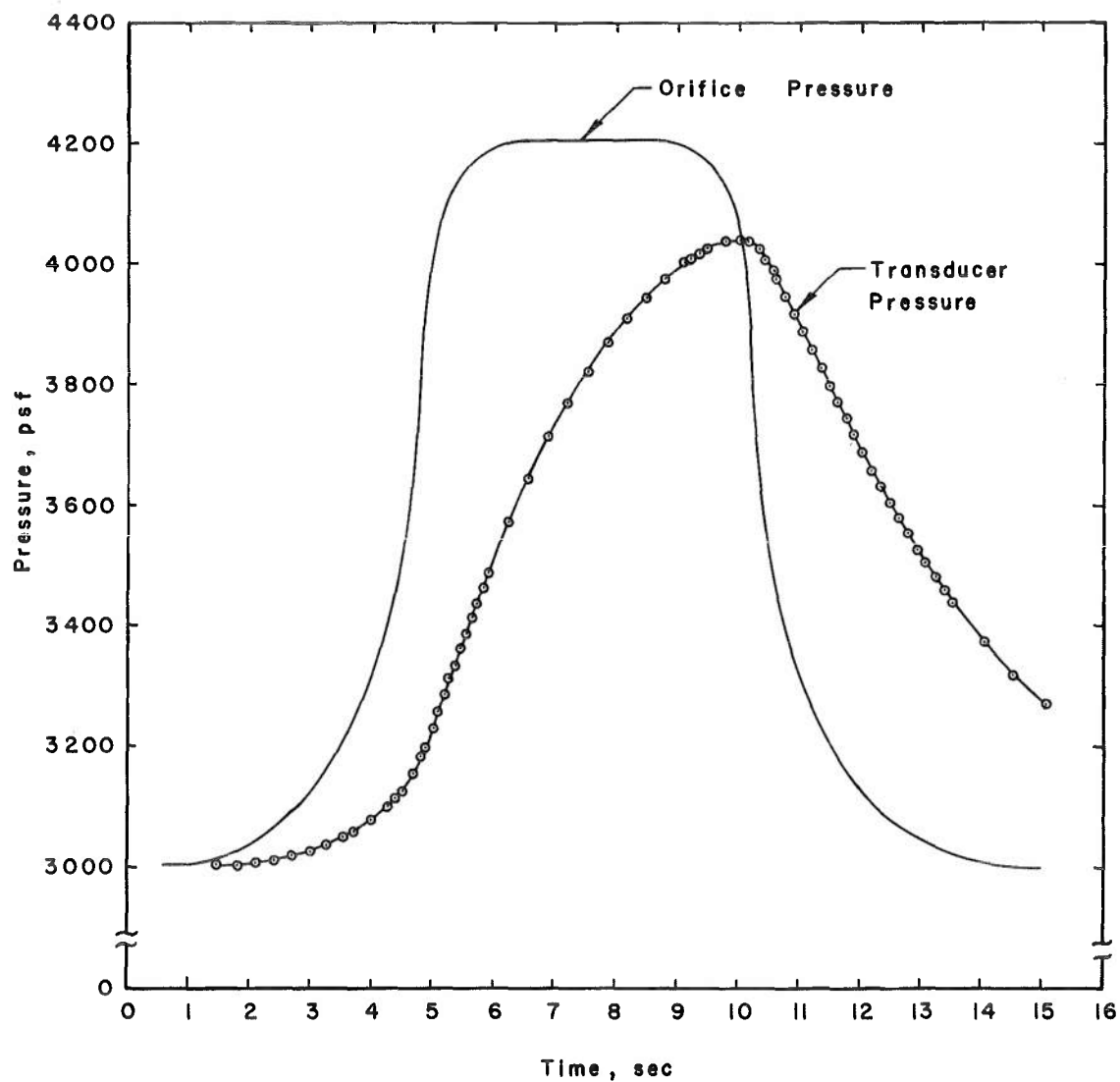


Fig. 18 Response of a Moving Pressure Probe to a Synthesized Nozzle Pressure Profile

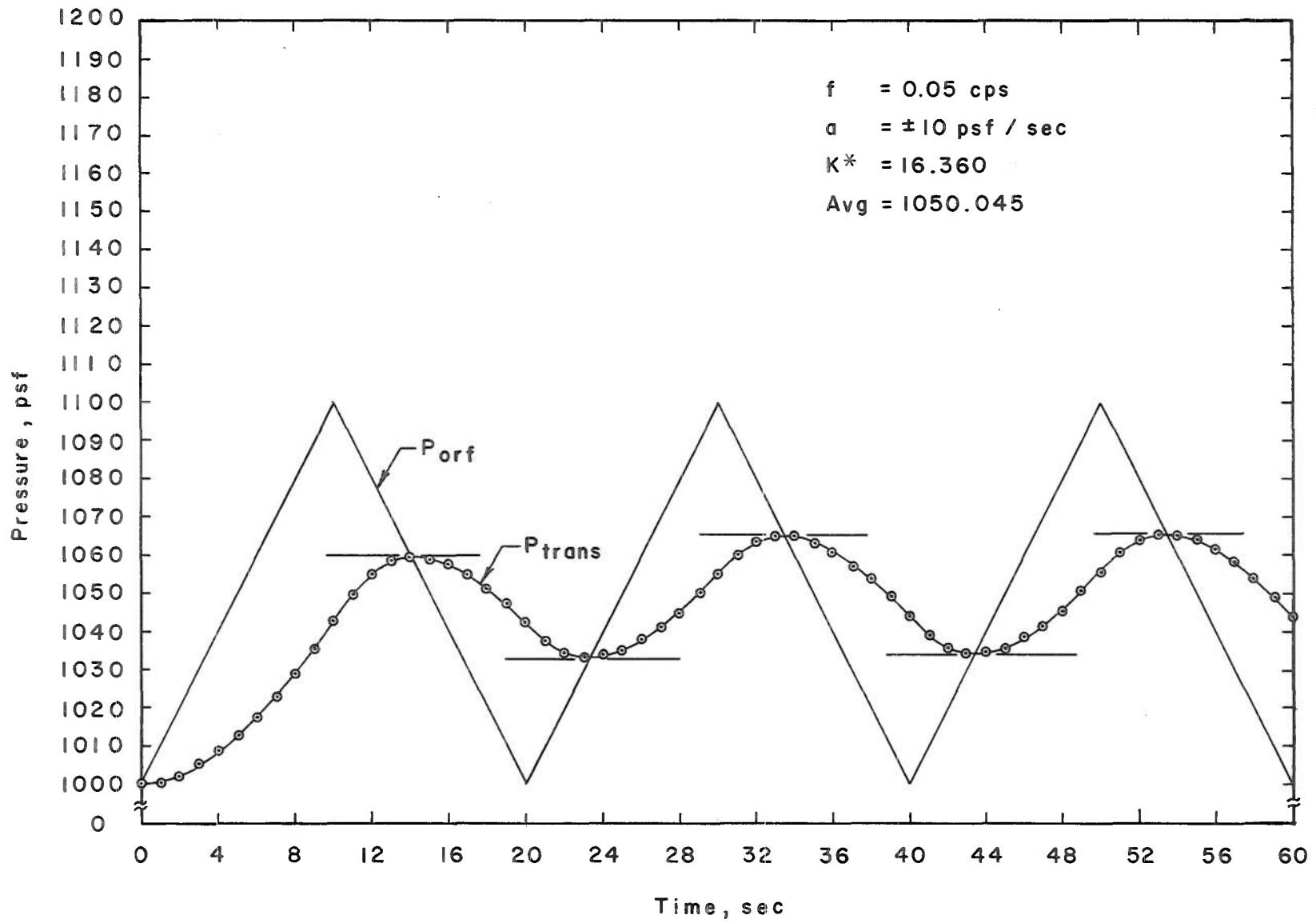


Fig. 19 Response to a Synthesized Triangular Wave

## APPENDIX II

### DERIVATION OF SYSTEM EQUATIONS

#### BASIC FLOW-PRESSURE EQUATIONS

The basic mass flow and pressure equations required for the development of the system equations are taken directly from Ref. 2 without explanation. If the flow is laminar ( $Re \leq 2000$ ) with negligible acceleration forces, mass flow is related to pressure by

$$w = - \frac{\partial p}{\partial x} P \left( \frac{A^2}{8\pi\mu RT_1} \right) \quad (\text{II-1})$$

For a single-tube system, the orifice pressure and transducer pressure are related by

$$(P_t^2 - P^2) \left( \frac{A^2}{16\pi\mu\ell} \right) = \frac{dP_t}{dt} \left( \frac{A\ell}{6} \right) + \frac{dP}{dt} \left( V + \frac{A\ell}{3} \right) \quad (\text{II-2})$$

The mass flow along the tube is

$$w = \frac{A}{RT_1} \left\{ \frac{x^2}{2\ell} \left( \frac{dP_t}{dt} - \frac{dP}{dt} \right) - x \frac{dP_t}{dt} - K \right\} \quad (\text{II-3})$$

and the mass flow at the orifice is given by

$$w_t = \frac{1}{RT_1} \left\{ V \frac{dP}{dt} + \frac{A\ell}{2} \left( \frac{dP_t}{dt} + \frac{dP}{dt} \right) \right\} \quad (\text{II-4})$$

#### THREE-TUBE EQUATION

Since the third tube and transducer can be considered as a single-tube system measuring the pressure  $P_{23}$  at the orifice, Eqs. (II-2) and (II-4) become

$$(P_{23}^2 - P^2) \left( \frac{A_3^2}{16\pi\mu\ell_3} \right) = \frac{dP_{23}}{dt} \left( \frac{A_3\ell_3}{6} \right) + \frac{dP}{dt} \left( V + \frac{A_3\ell_3}{3} \right) \quad (\text{II-5})$$

$$w_{23} = \frac{1}{RT_1} \left\{ V \frac{dP}{dt} + \frac{A_3\ell_3}{2} \left( \frac{dP_{23}}{dt} + \frac{dP}{dt} \right) \right\} \quad (\text{II-6})$$

Applying Eq. (II-3) to the second tube at  $x = \ell_2$ ,  $w_{23}$  is also obtained as

$$w_{23} = \frac{A_2}{RT_1} \left\{ \frac{\ell_2}{2} \left( \frac{dP_{12}}{dt} - \frac{dP_{23}}{dt} \right) - \ell_2 \frac{dP_{12}}{dt} - K \right\} \quad (\text{II-7})$$

Setting Eqs. (II-6) and (II-7) equal, solving for K and then substituting this expression for K in Eq. (II-3) with  $P_t = P_{12}$  and  $P = P_{23}$ , yields the mass flow in tube 2 as

$$w_2 = \frac{1}{RT_1} \left\{ \frac{A_2 x^2}{2\ell_2} \left( \frac{dP_{12}}{dt} - \frac{dP_{23}}{dt} \right) - A_2 x \frac{dP_{12}}{dt} + V \frac{dP}{dt} + \frac{A_2 \ell_2}{2} \left( \frac{dP_{12}}{dt} + \frac{dP_{23}}{dt} \right) + \frac{A_3 \ell_3}{2} \left( \frac{dP_{23}}{dt} + \frac{dP}{dt} \right) \right\} \quad (\text{II-8})$$

The mass flow in tube 2 can also be found from Eq. (II-1) as

$$w_2 = -\frac{\partial p}{\partial x} P \left( \frac{A_2^2}{8\pi\mu RT_1} \right) \quad (\text{II-9})$$

An equation for the square of the junction pressure  $P_{23}$  can now be found by:

1. Setting Eqs. (II-8) and (II-9) equal
2. Separating variables
3. Integrating from  $p = P_{12}$  at  $x = 0$  to  $p = P_{23}$  at  $x = \ell_2$
4. Solving for  $P_{23}^2$

The result is

$$P_{23}^2 = -\frac{16\pi\mu}{A_2^2} \left\{ \frac{A_2 \ell_2^2}{6} \left( \frac{dP_{12}}{dt} - \frac{dP_{23}}{dt} \right) - A_2 \ell_2 \frac{dP_{12}}{dt} + A_2 \ell_2^2 \frac{dP_{23}}{dt} + V \ell_2 \frac{dP}{dt} + \frac{A_3 \ell_3 \ell_2}{2} \left( \frac{dP_{23}}{dt} + \frac{dP}{dt} \right) \right\} + P_{12}^2 \quad (\text{II-10})$$

Solving for  $P_{23}^2$  in Eq. (II-10) and setting equal to Eq. (II-11) yields

$$P_{12}^2 - P^2 = 2K_2 \left\{ \left( \frac{A_2 \ell_2}{6} \right) \frac{dP_{12}}{dt} + \left( V + \frac{A_3 \ell_3}{2} \right) \frac{dP}{dt} + \left( \frac{A_2 \ell_2}{3} + \frac{A_3 \ell_3}{2} \right) \frac{dP_{23}}{dt} \right\} + 2K_3 \left\{ \left( V + \frac{A_3 \ell_3}{3} \right) \frac{dP}{dt} + \left( \frac{A_3 \ell_3}{6} \right) \frac{dP_{23}}{dt} \right\} \quad (\text{II-11})$$

where

$$K_2 = \frac{8\pi\mu\ell_2}{A_2^2} \quad K_3 = \frac{8\pi\mu\ell_3}{A_3^2} \quad (\text{II-12})$$

The derivative of  $P_{23}$  must now be eliminated. Substituting the average values of  $\partial p/\partial x$  for tubes 2 and 3 into Eq. (II-1), the tube mass flows are

$$w_2 = - \left( \frac{P_{12} - P_{23}}{\ell_2} \right) P \left( \frac{A_2^2}{8\pi\mu RT_1} \right) \quad (\text{II-13})$$

$$w_3 = - \left( \frac{P_{23} - P}{\ell_3} \right) P \left( \frac{A_3^2}{8\pi\mu R T_1} \right) \quad (\text{II-14})$$

At the junction of tubes 2 and 3, the pressure is  $P_{23}$  and  $w_2 = w_3$ , or

$$\left( \frac{P_{12} - P_{23}}{\ell_2} \right) A_2^2 = \left( \frac{P_{23} - P}{\ell_3} \right) A_3^2 \quad (\text{II-15})$$

Solving for  $P_{23}$

$$P_{23} = \frac{CP_{12} + P}{C + 1} \quad (\text{II-16})$$

where

$$C = \left( \frac{\ell_3}{\ell_2} \right) \left( \frac{A_2}{A_3} \right)^2 \quad (\text{II-17})$$

Differentiating Eq. (II-16)

$$\frac{dP_{23}}{dt} = \frac{1}{C+1} \left( C \frac{dP_{12}}{dt} + \frac{dP}{dt} \right) \quad (\text{II-18})$$

Substituting for  $\frac{dP_{23}}{dt}$  in Eq. (II-11) and collecting terms yields

$$P_{12}^2 - P^2 = K_a \frac{dP_{12}}{dt} + K_b \frac{dP}{dt} \quad (\text{II-19})$$

where

$$K_a = \frac{K_2}{3(C+1)} \left\{ A_2 \ell_2 (3C+1) + A_3 \ell_3 (C^2 + 3C) \right\} \quad (\text{II-20})$$

$$K_b = -\frac{2K_2}{3(C+1)} \left\{ A_2 \ell_2 + A_3 \ell_3 (C^2 + 3C + 3) + 3V(C+1)^2 \right\} \quad (\text{II-21})$$

Equation (II-19) is the differential equation relating the pressure at the transducer to the pressure at the junction of tubes 1 and 2. The intermediate pressure  $P_{23}$  and its derivative have been eliminated.

Elimination of pressure  $P_{12}$  is accomplished in basically the same manner in which  $P_{23}$  was eliminated. Therefore, the steps are only briefly outlined:

1. Find  $w_{12}$  from Eq. (II-8) by letting  $x = 0$ .
2. Find  $w_{12}$  by applying Eq. (II-3) to the first tube at  $x = \ell_1$ . Note that  $P_t = P_t$  and  $P = P_{12}$  for first tube.
3. Equate results of steps 1 and 2, solve for  $K$ , and substitute the resulting expression for  $K$  in Eq. (II-3) applied to first tube at  $x = x$ , resulting in an equation for  $w_1$ .

4. Find  $w_1$  by applying Eq. (II-1) to first tube.
5. Equate  $W_1$  expressions of steps 3 and 4, separate variables, and integrate from  $p = p$  at  $x = 0$  to  $p = p_{12}$  at  $x = \ell_1$ . Solve for  $P_{12}^2$ .
6. Solve Eq. (II-19) for  $P_{12}^2$ , set equal to  $P_{12}^2$  as found in step 5, and solve for  $P_t^2 - P^2$ . The pressure  $P_{12}$  is thus eliminated.
7. Set equation for  $w_1$  as found in step 4, with  $\frac{\delta p}{\delta x} = - \left( \frac{P_t - P_{12}}{\ell_1} \right)$ , equal to Eq. (II-13) at  $p = P_{12}$ . Solve for  $P_{12}$  as function of  $P_t$  and  $P_{23}$ . Differentiate  $P_{12}$  to find  $\frac{dP_{12}}{dt}$ . Substitute  $\frac{dP_{12}}{dt}$  expression for  $\frac{dP_{12}}{dt}$  in  $P_t^2 - P^2$  equation of step 6. The derivative of  $P_{12}$  is thus eliminated.
8. The derivative  $\frac{dP_{23}}{dt}$  reappears as the above steps are performed and must be eliminated from the  $P_t^2 - P^2$  equation resulting in step 7. Express  $\frac{dP_{23}}{dt}$  as a function of  $P_t$  and  $P$  by using Eq. (II-18), with  $\frac{dP_{12}}{dt}$  replaced by the  $\frac{dP_{12}}{dt}$  expression found in step 7. The derivative is thus eliminated.

The resulting equation, containing only the proper independent variable (applied orifice pressure), dependent variable (resulting transducer pressure) and system characteristics (geometric dimensions and fluid properties) is

$$P_t^2 - P^2 = K_T \frac{dP_t}{dt} + K_m \frac{dP}{dt} \quad (\text{II-22})$$

where

$$K_T = \frac{8\pi\mu}{3} \left\{ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{A_1^2} \right) [A_1 \ell_1 (3B+1) + A_2 \ell_2 (3B+3DC) + A_3 \ell_3 (3DC)] \right. \\ \left. + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{A_2^2} \right) [A_2 \ell_2 (3C+1) + A_3 \ell_3 (C^2+3C)] \right\} \quad (\text{II-23})$$

$$K_m = \frac{8\pi\mu}{3} \left\{ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{A_1^2} \right) [2A_1 \ell_1 + A_2 \ell_2 (3D+6) + A_3 \ell_3 (3D+3B+6) \right. \\ \left. + 6V(B+1)] + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{A_2^2} \right) [A_2 \ell_2 (3C+1) + A_3 \ell_3 (C^2+3C)] \right. \\ \left. + \left( \frac{2}{C+1} \right) \left( \frac{\ell_2}{A_2^2} \right) [A_2 \ell_2 + A_3 \ell_3 (C^2+3C+3) + 3V(C+1)^2] \right\} \quad (\text{II-24})$$



$$C = \left( \frac{\ell_3}{\ell_2} \right) \left( \frac{A_2}{A_3} \right)^2$$

$$D = \left( \frac{\ell_2}{\ell_1} \right) \left( \frac{A_1}{A_2} \right)^2 \quad (\text{II-25})$$

$$B = D(C + 1) \quad (\text{II-26})$$

Equation (II-22) is identical in form to Eq. (37) of Ref. 2. The geometric characteristics  $K_T$  and  $K_m$  given by Eqs. (II-23) and (II-24) are different in that they now contain the dimensions of a third tube. For a two-tube system ( $\ell_3 = 0$ ), Eqs. (II-23) and (II-24) become identical to the  $K_T$  and  $K_m$  equations of Ref. 2.

### SOLUTION OF DIFFERENTIAL EQUATION

Equation (II-22) can be rearranged to give

$$\frac{dP}{dt} + \frac{P^2}{K_m} = f(t) \quad (\text{II-27})$$

where the forcing function is given by

$$f(t) = \frac{P_t^2}{K_m} - \frac{K_T}{K_m} \frac{dP_t}{dt} \quad (\text{II-28})$$

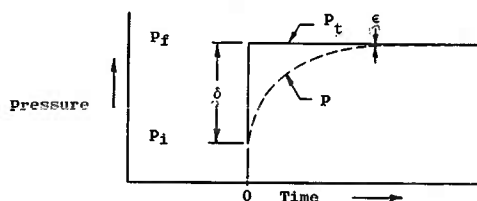
Equation (II-27) is a nonlinear, ordinary differential equation of first order and second degree with constant coefficients, commonly called a Riccati equation. The techniques normally employed for the solution of this equation depend to a great extent upon the form of the forcing function  $f(t)$ .

### STEP FUNCTION SOLUTION

For a step in pressure forcing function, as shown in Sketch 1,

$$\frac{dP_t}{dt} = 0 \text{ for } t > 0 \text{ and}$$

$$f(t) = \frac{P_t^2}{K_m} = \frac{(P_i + \delta)^2}{K_m} \quad (\text{II-29})$$



Sketch 1

The Riccati equation, Eq. (II-27), becomes

$$\frac{dP}{dt} + \frac{P^2}{K_m} = \frac{P_t^2}{K_m} \quad (\text{II-30})$$

If a particular solution,  $u$ , is known, the solution of this form of the Riccati equation reduces to the integration of a linear differential equation. The particular solution  $P = P_t = P_i$  at  $t = 0$  is known.

Letting

$$P = P_t + \frac{1}{u} \quad (\text{II-31})$$

then

$$u = \frac{1}{P - P_t} \quad (\text{II-32})$$

and

$$\frac{dP}{dt} = -\frac{1}{u^2} \frac{du}{dt} \quad (\text{II-33})$$

Substituting Eqs. (II-31) and (II-33) in Eq. (II-30) yields

$$-\frac{1}{u^2} \frac{du}{dt} + \frac{1}{K_m} \left( P_t + \frac{1}{u} \right)^2 = \frac{P_t^2}{K_m} \quad (\text{II-34})$$

or

$$\frac{du}{dt} = \frac{1}{K_m} \left\{ 2P_t u + 1 \right\} \quad (\text{II-35})$$

Separating variables

$$dt = K_m \left( \frac{du}{2P_t u + 1} \right) \quad (\text{II-36})$$

Now at  $t = 0$ ,  $P = P_i$  and  $u = 1/P_i - P_t$ , and at  $t = t$ ,  $P = P$  and  $u = 1/P - P_t$ . Integrating

$$\int_0^t dt = \frac{1}{P_i - P_t} \int_{\frac{1}{P_i - P_t}}^{\frac{1}{P - P_t}} \frac{du}{2P_t u + 1} \quad (\text{II-37})$$

results in the solution

$$t = \frac{K_m}{2P_t} \log_e \left\{ \left( \frac{P_t + P}{P_t - P} \right) \left( \frac{P_t - P_i}{P_t + P_i} \right) \right\} \quad (\text{II-38})$$

Equation (II-38) is so arranged to make time the dependent variable. Therefore, the lag time, or the time required for the transducer pressure to stabilize to within a specied error,  $\epsilon$ , following a step change in orifice pressure,  $\delta$ , can be expressed as

$$t_L = \frac{K_m}{2(P_i + \delta)} \log_e \left| \frac{2\delta(P_i + \delta) - \delta \epsilon}{\epsilon(2P_i + \delta)} \right| \quad (\text{II-39})$$

# RAMP FUNCTION SOLUTION

For a ramp in pressure forcing function, as shown in Sketch 2,

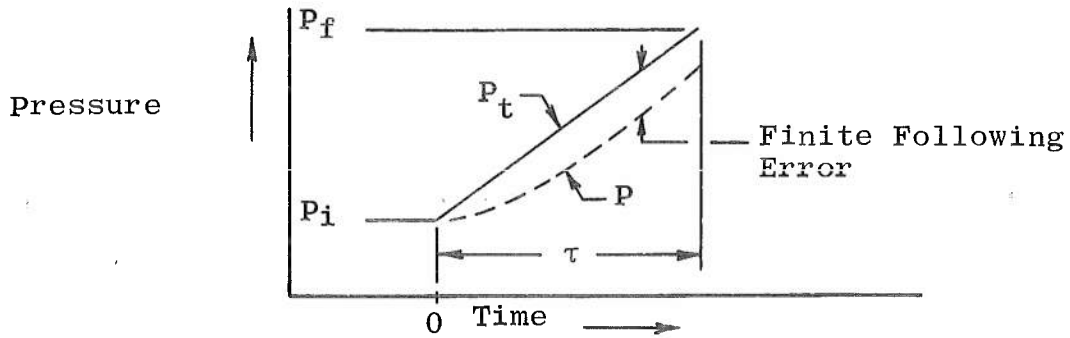
$$\frac{dP_t}{dt} = \frac{P_f - P_i}{\tau} \text{ for } t > 0 \text{ and } f(t) = kt^2 + mt + n \quad (\text{II-40})$$

where

$$k = \frac{1}{K_m} \left( \frac{P_f - P_i}{\tau} \right)^2 \quad (\text{II-41})$$

$$m = \frac{2}{K_m} \left\{ \frac{P_i(P_f - P_i)}{\tau} \right\} \quad (\text{II-42})$$

$$n = \frac{1}{K_m} P_i^2 - \left\{ \frac{K_T(P_f - P_i)}{\tau} \right\} \quad (\text{II-43})$$



Sketch 2

Through the use of a transformation, the first order nonlinear Riccati equation can be transformed into a second order linear differential equation that can be solved. Such a transformation is

$$y = e^{\int P g(t) dt} \quad (\text{II-44})$$

where  $g(t)$  is a function of the independent variable, time, that will make the differential equation linear.

Differentiating

$$\frac{dy}{dt} = e^{\int P g dt} \frac{d}{dt} \left[ \int P g dt \right] \quad (\text{II-45})$$

or

$$\frac{dy}{dt} = P g y \quad (\text{II-46})$$

or

$$P = \frac{1}{y g} \frac{dy}{dt} \quad (\text{II-47})$$

Differentiating Eq. (II-47)

$$\frac{dP}{dt} = \frac{1}{g} \frac{d}{dt} \left( \frac{1}{y} \frac{dy}{dt} \right) - \frac{1}{g^2} \left( \frac{1}{y} \frac{dy}{dt} \right) \frac{dg}{dt} \quad (\text{II-48})$$

Since

$$\begin{aligned} \frac{d}{dt} \left( \frac{1}{y} \frac{dy}{dt} \right) &= \frac{1}{y} \frac{d^2y}{dt^2} - \frac{1}{y^2} \frac{dy}{dt} \frac{dy}{dt} \\ &= \frac{1}{y} \frac{d^2y}{dt^2} - \left( \frac{1}{y} \frac{dy}{dt} \right)^2 \end{aligned} \quad (\text{II-49})$$

Eq. (II-48) becomes

$$\frac{dP}{dt} = \frac{1}{gy} \frac{d^2y}{dt^2} - \frac{1}{gy^2} \left( \frac{dy}{dt} \right)^2 - \frac{1}{g^2y} \frac{dy}{dt} \frac{dg}{dt} \quad (\text{II-50})$$

Substituting Eqs. (II-47) and (II-50) into Eq. (II-27) results in

$$\frac{1}{gy} \frac{d^2y}{dt^2} - \frac{1}{gy^2} \left( \frac{dy}{dt} \right)^2 - \frac{1}{g^2y} \frac{dy}{dt} \frac{dg}{dt} + \frac{1}{K_m g^2 y^2} \left( \frac{dy}{dt} \right)^2 = f(t) \quad (\text{II-51})$$

If  $g(t)$  is chosen to be equal to  $1/K_m$ , which makes  $\frac{dg}{dt} = 0$ , Eq. (II-51) reduces to the linear equation

$$\frac{d^2y}{dt^2} - \frac{f(t)y}{K_m} = 0 \quad (\text{II-52})$$

and the inverse transform is

$$P = \frac{K_m}{y} \frac{dy}{dt} \quad (\text{II-53})$$

If the  $mt$  term of the  $f(t)$  function is eliminated, Eq. (52) will be in the form of a differential equation for which a solution is known. Therefore, let

$$t = \frac{x}{(4k)^{1/4}} - \frac{m}{2k} \quad (\text{II-54})$$

therefore

$$dt = \frac{dx}{(4k)^{1/4}} \quad (\text{II-55})$$

Using the transformation,  $f(t)$  becomes

$$f(t) = f(x) = \frac{k^{1/2} x^2}{2} - \frac{m^2}{4k} + n \quad (\text{II-56})$$

Substitution of Eqs. (II-55) and (II-56) into Eq. (II-52) results in

$$\frac{d^2y}{dx^2} + \left[ \frac{x^2}{4K_m} - a \right] y = 0 \quad (\text{II-57})$$

where

$$a = \frac{m^2 - 4kn}{8K_m k^{3/2}} \quad (\text{II-58})$$

Equation (II-57) is a form of Weber's equation. Its solutions are parabolic cylinder or Weber-Hermite functions which can be found tabulated in Ref. 3 or found by the methods of Ref. 4. The solution is

$$y = C_1 W(a, x) + C_2 W(a, -x) \quad (\text{II-59})$$

where the Weber function is given by

$$W(a, x) = 2^{-3/4} \left\{ \sqrt{\frac{G_1}{G_3}} a_1 - \sqrt{\frac{2G_3}{G_1}} a_2 \right\} \quad (\text{II-60})$$

where

$$a_1 = 1 + \frac{ax^2}{2!} + (a_2 - 1/2) \frac{x^4}{4!} + (a^3 - 7/2a) \frac{x^6}{6!} + \dots \quad (\text{II-61})$$

$$a_2 = x + \frac{ax^3}{3!} + (a^2 - 3/2) \frac{x^5}{5!} + (a^3 - 13/2a) \frac{x^7}{7!} + \dots \quad (\text{II-62})$$

$$G_1 = |\Gamma(1/4 + i a/2)| \quad (\text{II-63})$$

$$G_3 = |\Gamma(3/4 + i a/2)| \quad (\text{II-64})$$

Therefore

$$P = K_m (4k)^{1/4} \left\{ \frac{C_1 W'(a, x) - C_2 W'(a, -x)}{C_1 W(a, x) + C_2 W(a, -x)} \right\} \quad (\text{II-65})$$

where

$$W'(a, x) = \frac{dW(a, x)}{dx} \quad (\text{II-66})$$

No loss in generality occurs if  $C_2 = 1$ , therefore

$$P = K_m (4k)^{1/4} \left\{ \frac{C_1 W'(a, x) - W'(a, -x)}{C_1 W(a, x) + W(a, -x)} \right\} \quad (\text{II-67})$$

### APPENDIX III STEP FORCING FUNCTION EQUATION LIST

#### AREAS

$$A_1 = 0.7854 d_1^2 \quad (\text{III-1})$$

$$A_2 = 0.7854 d_2^2 \quad (\text{III-2})$$

$$A_3 = 0.7854 d_3^2 \quad (\text{III-3})$$

#### DIMENSIONLESS CONSTANTS

$$C = \left( \frac{\ell_3}{\ell_2} \right) \left( \frac{A_2}{A_3} \right)^2 \quad C = 0 \text{ if } \ell_3 = 0 \quad (\text{III-4})$$

$$D = \left( \frac{\ell_2}{\ell_1} \right) \left( \frac{A_1}{A_2} \right)^2 \quad D = 0 \text{ if } \ell_2 = 0 \quad (\text{III-5})$$

$$B = D(C + 1) \quad (\text{III-6})$$

#### JUNCTION PRESSURES

$$P_{12} = \left( \frac{B}{B+1} \right) (P_i + \delta) + \left( \frac{1}{B+1} \right) P_i \quad (\text{III-7})$$

$$P_{23} = \left( \frac{DC}{B+1} \right) (P_i + \delta) + \left( \frac{D+1}{B+1} \right) P_i \quad (\text{III-8})$$

#### REYNOLDS NUMBER

$$Re_1 = \left| 1.1905 \times 10^5 \left( \frac{d_1^3}{\ell_1} \right) \left\{ (P_i + \delta)^2 - P_{12}^2 \right\} \right| \quad (\text{III-9})$$

$$Re_2 = \left| 1.1905 \times 10^5 \left( \frac{d_2^3}{\ell_2} \right) (P_{12}^2 - P_{23}^2) \right| \quad (\text{III-10})$$

$$Re_3 = \left| 1.1905 \times 10^5 \left( \frac{d_3^3}{\ell_3} \right) (P_{23}^2 - P_i^2) \right| \quad (\text{III-11})$$

## ACCELERATION CONDITIONS

$$a_1 = \left| \frac{24 (\ell_1/d_1)}{\text{Re}_1 \log_e \frac{P_{12}}{P_i + \delta}} \right| \quad (\text{III-12})$$

$$a_2 = \left| \frac{24 (\ell_2/d_2)}{\text{Re}_2 \log_e \frac{P_{23}}{P_{12}}} \right| \quad (\text{III-13})$$

$$a_3 = \left| \frac{24 (\ell_3/d_3)}{\text{Re}_3 \log_e \frac{P_i}{P_{23}}} \right| \quad (\text{III-14})$$

## GEOMETRIC CHARACTERISTICS

$$\begin{aligned} K_m = 5.16 \times 10^{-6} & \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{d_1^4} \right) \left\{ 2A_1 \ell_1 \right. \right. \\ & + A_2 \ell_2 (3D+6) + A_3 \ell_3 (3D+3B+6) + 6V(B+1) \left. \right\} \\ & + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 + A_3 \ell_3 (3C+1) + A_3 \ell_3 (C^2+3C) \right\} \\ & \left. + \left( \frac{2}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 + A_3 \ell_3 (C^2+3C+3) + 3V(C+1)^2 \right\} \right] \quad (\text{III-15}) \end{aligned}$$

$$K^* = K_m \cdot 10^{-3}$$

## LAG TIME

$$t_L = \frac{K_m}{2(P_i + \delta)} \log_e \left| \frac{2\delta(P_i + \delta) - \delta \epsilon}{\epsilon (2P_i + \delta)} \right| \quad (\text{III-16})$$

Test: If  $P_i + \delta < 0$

then  $t_L = 0$

## UNIT CONVERSIONS

$$\epsilon = \frac{\epsilon (\text{Percent Final Value})}{100} \cdot P_f \quad \text{Options 2 and 4} \quad (\text{III-17})$$

$$\delta = \frac{\delta (\text{Percent Initial Value})}{100} \cdot P_i \quad \text{Options 3 and 4} \quad (\text{III-18})$$

**OPTIMIZATION STARTING VALUE (OPTION 6)**

$$\left\{ (d_2)_{\text{opt}} \right\}_{\text{approx}} = d_1 \left( \frac{\ell_2}{2\ell_1} \right)^{1/4} \text{ Two Tube} \quad (\text{III-19})$$

$$\left\{ (d_2)_{\text{opt}} \right\}_{\text{approx}} = d_1 \left( \frac{\ell_2 + \ell_3}{2\ell_1} \right)^{1/4} \text{ Three Tube} \quad (\text{III-20})$$

**TEMPERATURE**

$$T_1 = T + 460 \quad (\text{III-21})$$



# APPENDIX IV RAMP FORCING FUNCTION EQUATION LIST

## AREAS

$$A_1 = 0.7854 d_1^2 \quad (\text{IV-1})$$

$$A_2 = 0.7854 d_2^2 \quad (\text{IV-2})$$

$$A_3 = 0.7854 d_3^2 \quad (\text{IV-3})$$

## DIMENSIONLESS CONSTANTS

$$C = \left( \frac{\ell_3}{\ell_2} \right) \left( \frac{A_2}{A_3} \right)^2 \quad C = 0 \text{ if } \ell_3 = 0 \quad (\text{IV-4})$$

$$D = \left( \frac{\ell_2}{\ell_1} \right) \left( \frac{A_1}{A_2} \right)^2 \quad D = 0 \text{ if } \ell_2 = 0 \quad (\text{IV-5})$$

$$B = D(C + 1) \quad (\text{IV-6})$$

## JUNCTION PRESSURES

$$P_{12} = \left( \frac{B}{B+1} \right) (P_i + P_f - P_o) + \left( \frac{1}{B+1} \right) P_i \quad (\text{IV-7})$$

$$P_{23} = \left( \frac{DC}{B+1} \right) (P_i + P_f - P_o) + \left( \frac{D+1}{B+1} \right) P_i \quad (\text{IV-8})$$

## REYNOLDS NUMBER

$$Re_1 = \left| 1.1905 \times 10^5 \left( \frac{d_1^3}{\ell_1} \right) \left\{ (P_i + P_f - P_o)^2 - P_{12}^2 \right\} \right| \quad (\text{IV-9})$$

$$Re_2 = \left| 1.1905 \times 10^5 \left( \frac{d_2^3}{\ell_2} \right) (P_{12}^2 - P_{23}^2) \right| \quad (\text{IV-10})$$

$$Re_3 = \left| 1.1905 \times 10^5 \left( \frac{d_3^3}{\ell_3} \right) (P_{23}^2 - P_o^2) \right| \quad (\text{IV-11})$$

## ACCELERATION CONDITIONS

$$a_1 = \left| \frac{24 (\ell_1/d_1)}{\text{Re}_1 \log_e \left( \frac{P_{12}}{P_i + P_f - P_o} \right)} \right| \quad (\text{IV-12})$$

$$a_2 = \left| \frac{24 (\ell_2/d_2)}{\text{Re}_2 \log_e \left( \frac{P_{23}}{P_{12}} \right)} \right| \quad (\text{IV-13})$$

$$a_3 = \left| \frac{24 (\ell_3/d_3)}{\text{Re}_3 \log_e \left( \frac{P_o}{P_{23}} \right)} \right| \quad (\text{IV-14})$$

## TEMPERATURE

$$T_1 = T + 460 \quad (\text{IV-15})$$

## GEOMETRIC CHARACTERISTICS

$$\begin{aligned} K_m = 5.16 \times 10^{-6} & \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{d_1^4} \right) \left\{ 2A_1 \ell_1 \right. \right. \\ & + A_2 \ell_2 (3D+6) + A_3 \ell_3 (3D+3B+6) + 6V(B+1) \left. \right\} \\ & + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 (3C+1) + A_3 \ell_3 (C^2+3C) \right\} \\ & + \left( \frac{2}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 + A_3 \ell_3 (C^2+3C+3) + 3V(C+1)^2 \right\} \left. \right] \quad (\text{IV-16}) \end{aligned}$$

$$\begin{aligned} K_T = 5.16 \times 10^{-6} & \left( \frac{723}{T_1 + 198.6} \right) \left( \frac{T_1}{524.4} \right)^{1.5} \left[ \left( \frac{1}{B+1} \right) \left( \frac{\ell_1}{d_1^4} \right) \left\{ A_1 \ell_1 (3B+1) \right. \right. \\ & + A_2 \ell_2 (3B+3DC) + A_3 \ell_3 (3DC) \left. \right\} + \left( \frac{1}{B+1} \right) \left( \frac{1}{C+1} \right) \left( \frac{\ell_2}{d_2^4} \right) \left\{ A_2 \ell_2 (3C+1) \right. \\ & + A_3 \ell_3 (C^2+3C) \left. \right\} \left. \right] \quad (\text{IV-17}) \end{aligned}$$

$$K^* = K_m \cdot 10^{-3} \quad (\text{IV-18})$$

**FORCING FUNCTION COEFFICIENTS**

$$k = \frac{1}{K_m} \left( \frac{P_f - P_i}{\tau} \right)^2 \quad (\text{IV-19})$$

$$m = \frac{2}{K_m} \left\{ \frac{P_i (P_f - P_i)}{\tau} \right\} \quad (\text{IV-20})$$

$$n = \frac{1}{K_m} \left\{ P_i^2 - \frac{K_T (P_f - P_i)}{\tau} \right\} \quad (\text{IV-21})$$

**DIFFERENTIAL EQUATION**

$$\frac{dP}{dt} + \frac{P^2}{K_m} = kt^2 + mt + n \quad (\text{IV-22})$$

**INTEGRATION**

$$\text{Initial Conditions: } P = P_o \text{ at } t = 0 \quad (\text{IV-23})$$

$$dP/dt = 0 \text{ at } t = 0 \text{ if } P_o = P_i \quad (\text{IV-24})$$

$$\text{Integration Increment: } \Delta t = 0.0002\tau \quad (\text{IV-25})$$

$$\text{Printout Increment: } t = 0.1\tau, 0.2\tau, \dots 1.0\tau \quad (\text{IV-26})$$



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14.

## KEY WORDS

theoretical equations

pressure measurements

digital computers

step functions

ramp forcing functions

cylindrical tubing

laminar flow

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